From: The Mobile Radio Propagation Channel
by David Parsons (1996)

2

Fundamentals of VHF and UHF propagation

2.1 INTRODUCTION

Having established the suitability of the VHF and UHF bands for mobile communications, we can now proceed to develop some fundamental relationships between the transmitted and received power, distance (range) and carrier frequency. Firstly though, we state a few relevant definitions which will aid an understanding of what follows.

At frequencies below 1 GHz antennas normally consist of a wire or wires of a suitable length coupled to the transmitter via a transmission line. At these frequencies it is relatively easy to design an assembly of wire radiators which form an array, in order to beam the radiation in a particular direction. For distances large in comparison with the wavelength and the dimensions of the array, the field strength in free space decreases with an increase in distance, and a plot of the field strength as a function of spatial angle is known as the radiation pattern of the antenna.

Antennas can be designed to have radiation patterns which are not omnidirectional, and it is convenient to have a figure of merit to quantify the ability of the antenna to concentrate the radiated energy in a particular direction. The directivity D of an antenna is defined as

$$D = \frac{\text{Power density at a distance } d \text{ in the direction of maximum radiation}}{\text{mean power density at distance } d}$$

This is a measure of the extent to which the power density in the direction of maximum radiation exceeds the average power density at the same distance. The directivity involves knowing the power actually transmitted by the antenna and this differs from the power supplied at the terminals by the losses in the antenna itself. From the system designer's point of view it is more convenient to work in terms of terminal power and a power gain G can be defined as

$$G = \frac{\text{Power density at a distance } d \text{ in the direction of maximum radiation}}{P_T/4\pi d^2}$$

where P_T = power supplied to the antenna.

So, given P_T and G it is possible to calculate the power density at any

point in the far field that lies in the direction of maximum radiation. A knowledge of the radiation pattern is necessary to determine fields at other points.

The power gain is unity for an isotropic antenna, i.e. one which radiates uniformly in all directions, and an alternative definition of power gain is therefore the ratio of power density in the direction of maximum radiation to that of an isotropic antenna radiating the same power. We can calculate the power gain of an antenna or an array by integrating the outward power flow and relating it to the mean power density. As an example, the power gain of a $\lambda/2$ dipole is 1.64 (2.15 dB) in a direction normal to the dipole and is the same whether the antenna is used for transmission or reception.

There is a concept known as effective area which is useful when dealing with antennas in the receiving mode. If an antenna is placed in the field of an electromagnetic wave the received power available at its terminals is the power per unit area carried by the wave × effective area, i.e. $P = WA$. It can be shown [1, Chap 11] that the effective area of an antenna and its power gain are related by

$$A = \frac{\lambda^2 G}{4\pi} \tag{2.1}$$

2.2 PROPAGATION IN FREE SPACE

Radio propagation is a subject where deterministic analysis can only be applied in a few, rather simple cases. The extent to which these cases represent practical conditions is a matter for individual interpretation, but they do give an insight into the basic propagation mechanisms and establish bounds.

If a transmitting antenna is located in free space, i.e. remote from the earth or any obstructions, then if it has a gain G_T in the direction to a receiving antenna, the power density (i.e. power per unit area) at a distance (range) d in the chosen direction is

$$W = \frac{P_T G_T}{4\pi d^2} \tag{2.2}$$

The available power at the receiving antenna, which has an effective area A is therefore

$$P_R = \frac{P_T G_T}{4\pi d^2} \cdot A$$
$$= \frac{P_T G_T}{4\pi d^2} \cdot \frac{\lambda^2 G_R}{4\pi}$$

where G_R is the gain of the receiving antenna.

Thus, we obtain

$$\frac{P_R}{P_T} = G_T G_R \left[\frac{\lambda}{4\pi d}\right]^2 \tag{2.3}$$

which is a fundamental relationship known as the free-space or Friis equation [2]. The well known relationship between wavelength λ, frequency f and velocity of propagation c ($c = f\lambda$) can be used to write this equation in the alternative form

$$\frac{P_R}{P_T} = G_T G_R \left[\frac{c}{4\pi f d}\right]^2 \tag{2.4}$$

The propagation loss (or path loss) is conveniently expressed in dB and from eqn. (2.4) we can write

$$L_F = 10\log_{10}\frac{P_R}{P_T}$$
$$= 10\log_{10}G_T + 10\log_{10}G_R - 20\log_{10}f - 20\log_{10}d + k \tag{2.5}$$

where

$$k = 20\log_{10}\frac{3\times 10^8}{4\pi} = 147.6$$

It is often useful to compare path loss with the basic path loss L_B between isotropic antennas, which is

$$L_B(\text{dB}) = -32.44 - 20\log_{10}f_{\text{MHz}} - 20\log_{10}d_{\text{km}} \tag{2.6}$$

Equation (2.4) shows that free-space propagation obeys an inverse square law with range d, so that the received power falls by 6 dB when the range is doubled (or reduces by 20 dB per decade). Similarly the path loss increases with the square of the transmission frequency, so that losses also increase by 6 dB if the frequency is doubled. High gain antennas can be used to make up for this loss and fortunately such antennas are relatively easily designed at frequencies in and above the VHF band. This provides a solution for fixed (point-to-point) links, but not for VHF and UHF mobile links where omnidirectional coverage is required.

In some cases it is convenient to write an expression for the electric field strength at a known distance from a transmitting antenna rather than the power density. This can be done by noting that the relationship between field strength and power density is

$$W = \frac{E^2}{\eta}$$

where η is the characteristic wave impedance of free space. Its value is 120π ($\sim 377\Omega$) and so eqn. (2.2) can be written

$$\frac{E^2}{120\pi} = \frac{P_T G_T}{4\pi d^2}$$

giving

$$E = \frac{\sqrt{30 P_T G_T}}{d} \tag{2.7}$$

Finally, we note that the maximum useful power that can be delivered to the terminals of a matched receiver is

$$P = \frac{E^2 A}{\eta} = \frac{E^2}{120\pi}\cdot\frac{\lambda^2 G_R}{4\pi} = \left(\frac{E\lambda}{2\pi}\right)^2 \frac{G_R}{120} \tag{2.8}$$

2.3 PROPAGATION OVER A REFLECTING SURFACE

The free-space propagation equation applies only under very restricted conditions; in practical situations there are almost always obstructions in or near the propagation path or surfaces from which the radio waves can be reflected. A very simple case, but one of practical interest, is that of propagation between two elevated antennas within line of sight of each other, above the surface of the earth. We will consider two cases, firstly that of propagation over a spherical reflecting surface and secondly when the distance between the antennas is small enough for us to neglect curvature and assume the reflecting surface to be flat. In these cases, illustrated in Figs. 2.1 and 2.4, the received signal is made up of a combination of direct and ground-reflected waves. In order to determine the resultant we need to know the reflection coefficient.

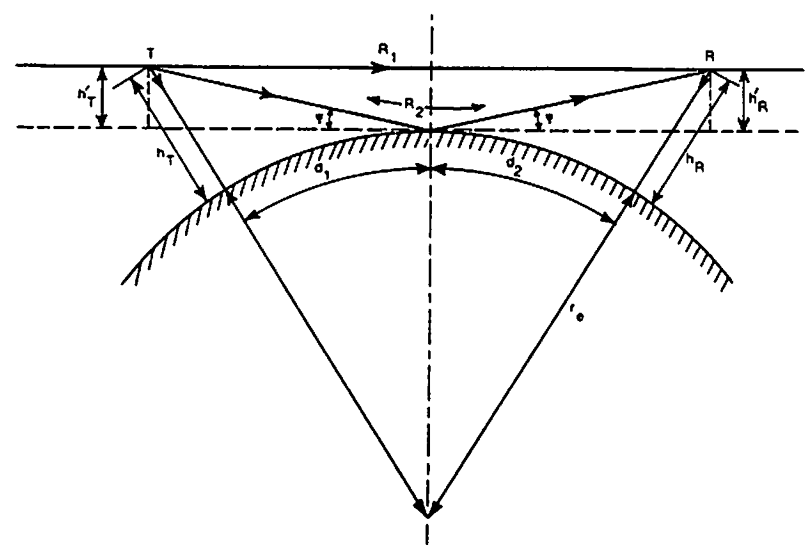

Fig. 2.1 Two mutually-visible antennas located above a smooth, spherical earth of effective radius r_e

2.3.1 The reflection coefficient of the earth

The amplitude and phase of the ground-reflected wave depends on the reflection coefficient of the earth at the point of reflection and differs for horizontal and vertical polarisation. In practice the earth is neither a perfect conductor nor a perfect dielectric so the reflection coefficient depends on the ground-constants, in particular and dielectric constant ε and the conductivity σ.

For a horizontally-polarised wave incident on the surface of the earth (assumed to be perfectly smooth) the reflection coefficient is given by [1, Chap. 16]

$$\rho_h = \frac{\sin\psi - \sqrt{(\varepsilon/\varepsilon_0 - j\sigma/\omega\varepsilon_0) - \cos^2\psi}}{\sin\psi + \sqrt{(\varepsilon/\varepsilon_0 - j\sigma/\omega\varepsilon_0) - \cos^2\psi}}$$

where ω is the angular frequency of the transmission and ε_o is the dielectric constant of free space. Writing ε_r as the relative dielectric constant of the earth yields

$$\rho_h = \frac{\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}{\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}} \qquad (2.9)$$

where

$$x = \frac{\sigma}{\omega\varepsilon_0} = \frac{18 \times 10^9\sigma}{f}$$

For vertical polarisation the corresponding expression is

$$\rho_v = \frac{(\varepsilon_r - jx)\sin\psi - \sqrt{(\varepsilon_r - jx) - \cos^2\psi}}{(\varepsilon_r - jx)\sin\psi + \sqrt{(\varepsilon_r - jx) - \cos^2\psi}} \qquad (2.10)$$

It is apparent that the reflection coefficients ρ_h and ρ_v are complex and the reflected wave will therefore differ in both magnitude and phase from the incident wave. Examination of eqns. (2.9) and (2.10) reveals some quite interesting differences. For horizontal polarisation the relative phase of the incident and reflected waves is nearly 180° for all angles of incidence. For very small values of ψ (near-grazing incidence) eqn. (2.9) shows that the reflected wave is equal in magnitude and 180° out of phase with the incident wave for all frequencies and all ground conductivities. In other words, for grazing incidence

$$\rho_h = |\rho_h| \angle\theta = 1 \angle 180^\circ = -1 \qquad (2.11)$$

As the angle of incidence is increased then $|\rho_h|$ and θ change, but only by relatively small amounts. The change is greatest at higher frequencies and when the ground conductivity is poor.

For vertical polarisation the results are quite different. At grazing incidence, there is no difference between horizontal and vertical polarisation and eqn. (2.11) still applies. However as ψ is increased, substantial differences appear.

The magnitude and relative phase of the reflected wave both decrease rapidly as ψ increases and at an angle known as the pseudo-Brewster angle the magnitude becomes a minimum and the phase reaches $-90°$. At values of ψ greater than the Brewster angle, $|\rho_v|$ increase again and the phase tends towards zero. The very sharp changes that occur in these circumstances are illustrated by Fig. 2.2 which shows the values of $|\rho_v|$ and θ as functions of the angle of incidence ψ. It can be seen that the pseudo-Brewster angle is about 15° at frequencies that are of interest for mobile communications ($x \ll \varepsilon_r$) although at lower frequencies and higher conductivities it becomes smaller, approaching zero if $x \gg \varepsilon_r$.

Table 2.1 shows typical values for the ground constants that affect the value of ρ.

It can be seen that the conductivity of flat, good ground is much higher than for the poorer ground found in mountainous areas, whilst the dielectric constant, typically 15, can be as low as 4 or as high as 30. It should be noted that over lakes or seas, the reflection properties are quite different because

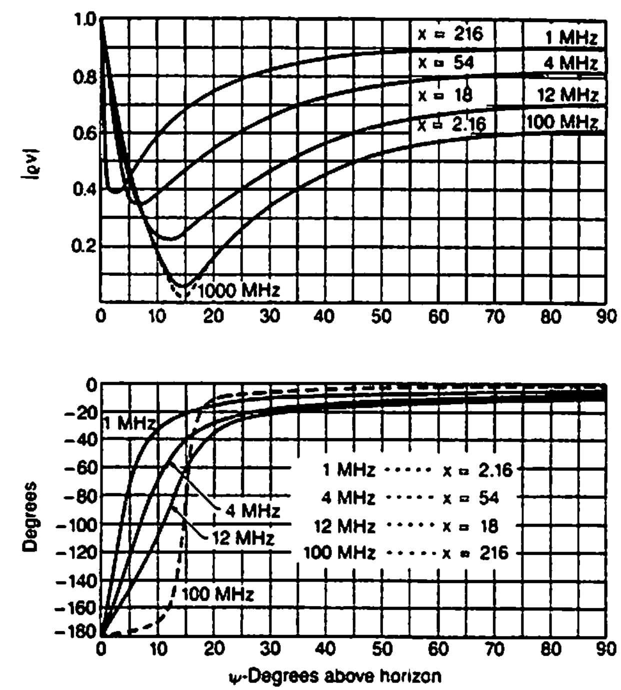

Fig. 2.2 Magnitude and phase of the plane wave reflection coefficient for vertical polarisation. Curves drawn for $\sigma = 12 \times 10^{-3}$, $\varepsilon_r = 15$. Approximate results for other frequencies and conductivities can be obtained by calculating the value of x as $18 \times 10^3\sigma/f_{MHz}$

Table 2.1 TYPICAL VALUES OF GROUND CONSTANTS

Surface	*Conductivity* σ *(siemens)*	*Dielectric constant* ε_r
Poor ground (dry)	10^{-3}	4–7
Average ground	5×10^{-3}	15
Good ground (wet)	2×10^{-2}	25–30
Sea water	5	81
Fresh water	10^{-2}	81

of the high values of both σ and ε_r. Eqn. (2.11) applies for horizontal polarisation, particularly over seawater, but ρ may be significantly different from -1 for vertical polarisation.

2.3.2 Propagation over a curved reflecting surface

The situation of two mutually-visible antennas sited on a smooth earth of effective radius r_e is shown in Fig. 2.1. The heights of the antennas above the earth's surface are h_T and h_R and above the tangent plane through the point of reflection the heights are h'_T and h'_R. Simply geometry gives,

$$d_1^2=[r_e+(h_T-h'_T)]^2-r_e^2=(h_T-h'_T)^2+2r_e(h_T-h'_T)\simeq 2r_e(h_T-h'_T) \quad (2.12)$$

and similarly

$$d_2^2\simeq 2r_e(h_R-h'_R) \quad (2.13)$$

Using eqns. (2.12) and (2.13) we obtain

$$h'_T=h_T-\frac{d_1^2}{2r_e} \quad \text{and} \quad h'_R=h_R-\frac{d_2^2}{2r_e} \quad (2.14)$$

The reflecting point, where the two angles marked ψ are equal, can be determined by noting that, providing $d_1, d_2 \gg h_T, h_R$, the angle ψ in radians, is given by

$$\psi\simeq\frac{h'_T}{d_1}=\frac{h'_R}{d_2}$$

Hence,

$$\frac{h'_T}{h'_R}\simeq\frac{d_1}{d_2} \quad (2.15)$$

Using the obvious relationship $d=d_1+d_2$ together with eqns. (2.14) and (2.15) allows us to formulate a cubic equation in d_1,

$$2d_1^3-3dd_1^2+[d^2-2r_e(h_T+h_R)]d_1+2r_eh_Td=0 \quad (2.16)$$

The appropriate root of this equation can be found by standard methods

starting from the rough approximation

$$d_1\simeq\frac{d}{1+h_T/h_R}$$

In order to calculate the field strength at a receiving point it is normally assumed that the difference in path length between the direct and ground reflected waves is negligible as far as attenuation is concerned, but it cannot be neglected with regard to the phase difference along the two paths. The length of the direct path is

$$R_1=d\left[1+\frac{(h'_T-h'_R)^2}{d^2}\right]^{1/2}$$

whilst the length of the reflected path is

$$R_2=d\left[1+\frac{(h'_T+h'_R)^2}{d^2}\right]^{1/2}$$

The difference $\Delta R=R_2-R_1$ is

$$\Delta R=d\left\{\left[1+\frac{(h'_T+h'_R)^2}{d^2}\right]^{1/2}-\left[1+\frac{(h'_T-h'_R)^2}{d^2}\right]^{1/2}\right\}$$

and if $d\gg h'_T, h'_R$ this reduces to

$$\Delta R=\frac{2h'_Th'_R}{d} \quad (2.17)$$

The corresponding phase difference is

$$\Delta\phi=\frac{2\pi}{\lambda}\Delta R=\frac{4\pi h'_Th'_R}{\lambda d} \quad (2.18)$$

If the field strength at the receiving antenna due to the direct wave is E_d, then the total received field E is

$$E=E_d[1+\rho\exp(-j\Delta\phi)]$$

where ρ is the reflection coefficient of the earth, and $\rho=|\rho|\exp(j\theta)$. Thus,

$$E=E_d\{1+|\rho|\exp[-j(\Delta\phi-\theta)]\} \quad (2.19)$$

This equation can be used to calculate the received field strength at any location, but it should be noted that the curvature of the spherical earth produces a certain amount of divergence of the ground-reflected wave as shown in Fig. 2.3. This effect can be taken into account by using, in eqn. (2.19) a value of ρ which is different from that derived in Section (2.3.1) for reflection from a plane surface. The appropriate modification consists of multiplying the value of ρ for a plane surface by a divergence factor D, given by [3],

$$D\simeq\left[1+\frac{2d_1d_2}{r_e(h'_T+h'_R)}\right]^{-1/2} \quad (2.20)$$

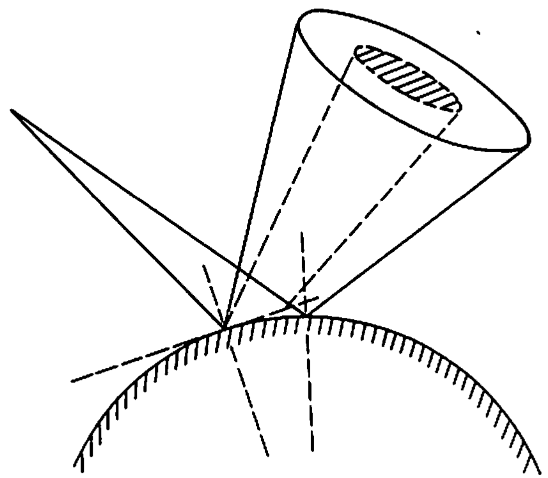

Fig. 2.3 Divergence of reflected rays from a spherical surface

The value of D can be of the order of 0.5, so the effect of the ground-reflected wave is considerably reduced.

2.3.3 Propagation over a plane reflecting surface

For distances less than a few tens of kilometres, it is often permissible to neglect earth curvature and assume the surface to be smooth and flat as in Fig. 2.4. If, in addition, we assume grazing incidence so that $\rho = -1$, then eqn. (2.19) becomes

$$E = E_d[1 - \exp(-j\Delta\phi)]$$
$$= E_d[1 - \cos\Delta\phi + j\sin\Delta\phi]$$

Thus,

$$|E| = |E_d|\,[1 + \cos^2\Delta\phi - 2\cos\Delta\phi + \sin^2\Delta\phi]^{1/2}$$
$$= 2|E_d|\sin\frac{\Delta\phi}{2}$$

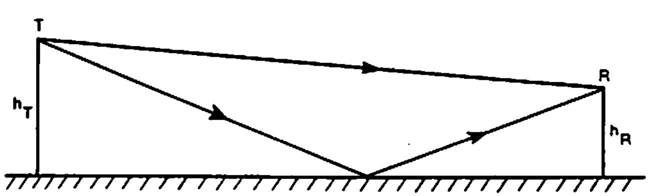

Fig. 2.4 Propagation over a plane earth

and using eqn. (2.18), with $h'_T = h_T$ and $h'_R = h_R$

$$|E| = 2|E_d|\sin\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

The received power P_R is proportional to E^2 so

$$P_R = 4|E_d|\,sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right)$$

$$= 4P_T\left(\frac{\lambda}{4\pi d}\right)^2 G_T G_R \sin^2\left(\frac{2\pi h_T h_R}{\lambda d}\right) \quad (2.21)$$

If $d \gg h_T$, h_R, this becomes

$$\frac{P_R}{P_T} = G_T G_R\left(\frac{h_T h_R}{d^2}\right)^2 \quad (2.22)$$

Eqn. (2.22) is known as the plane-earth propagation equation. It differs from the free-space relationship, eqn. (2.3), in two important ways. First, as a consequence of the assumption that $d \gg h_T$, h_R the angle $\Delta\phi$ is small and λ cancels out of eqn. (2.22) leaving it frequency-independent. Secondly, it shows an inverse fourth-power law with range rather than the inverse square law of eqn. (2.3). This means a far more rapid decrease in received power with range, 12 dB for each doubling of distance in this case.

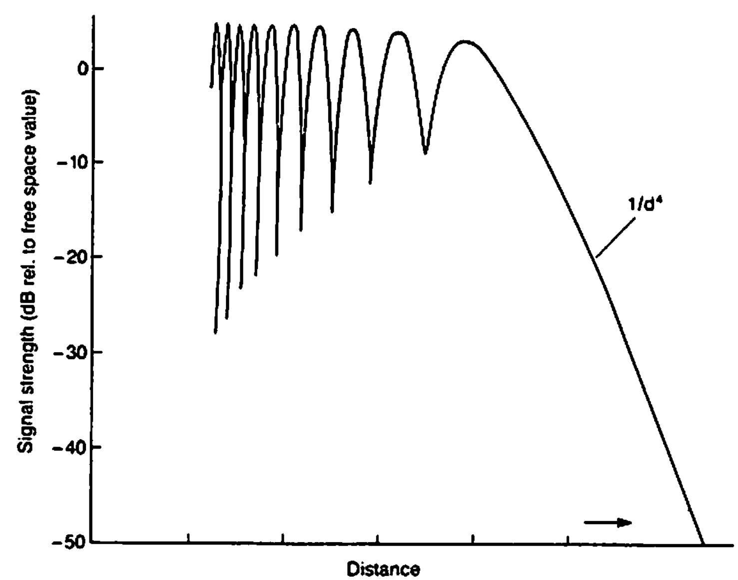

Fig. 2.5 Variation of signal strength with distance in the presence of a specular reflection.

It should be emphasised that eqn. (2.22) only applies at ranges where the assumption $d \gg h_T$, h_R is valid. Close to the transmitter eqn. (2.21) must be used and this gives alternate maxima and minima in the signal strength as shown in Fig. 2.5.

In convenient logarithmic form eqn. (2.22) can be written

$$L_P = 10\log_{10}G_T + 10\log_{10}G_R + 20\log_{10}h_T + 20\log_{10}h_R - 40\log_{10}d \tag{2.23}$$

and by comparison with eqn. (2.6) we can write a 'basic loss' between isotropic antennas as

$$L_B = 20\log_{10}h_T + 20\log_{10}h_R - 40\log_{10}d \tag{2.24}$$

2.4 GROUND ROUGHNESS

The discussion in the previous section pre-supposed that the reflecting surface is smooth and the analysis was therefore based on the assumption that a specular reflection takes places at the point where the transmitted wave is incident on the earth's surface. When the surface is rough the specular reflection assumption is no longer realistic since a rough surface presents many facets to the incident wave. A diffuse reflection therefore occurs and the mechanism is more akin to scattering. In these conditions characterisation by a single, complex reflection coefficient is not appropriate since the random nature of the surface results in an unpredictable situation. Only a small fraction of the incident energy may be scattered in the direction of the receiving antenna, and the 'ground-reflected' wave may therefore make a negligible contribution to the received signal.

The question therefore arises as to what constitutes a 'rough' as opposed to a smooth surface. Clearly a surface that might be considered rough at some frequencies and angles of incidence may approach a smooth surface if these parameters are changed. A measure of roughness is needed to quantify the problem and the criterion normally used is known as the Rayleigh criterion. The problem is illustrated in Fig. 2.6 (a) and an idealised rough surface profile is shown in Fig. 2.6 (b).

Consider the two rays A and B shown in Fig. 2.6 (b). Ray A is reflected from the upper part of the rough surface and ray B from the lower part. Relative to the wavefront AA′ shown, the difference in path length of the two rays when they reach the points C and C′ after reflection is

$$\begin{aligned}\Delta l &= (AB + BC) - (A'B' + B'C') \\ &= \frac{d}{\sin\psi}(1-\cos 2\psi) \\ &= 2d\sin\psi \end{aligned} \tag{2.25}$$

The phase difference between C and C′ is therefore

$$\Delta\theta = \frac{2\pi}{\lambda}\Delta l = \frac{4\pi d\sin\psi}{\lambda} \tag{2.26}$$

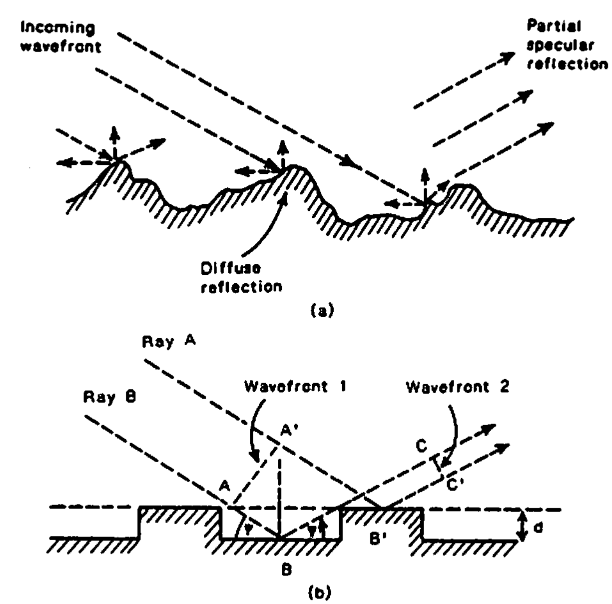

Fig. 2.6 Reflection from a semi-rough surface: (a) practical terrain situation, (b) idealised model

If the height d is small in compaison with λ then the phase difference $\Delta\theta$ is also small. For practical purposes a specular reflection appears to have occurred and the surface therefore seems to be smooth. On the other hand, extreme roughness corresponds to $\Delta\theta = \pi$ i.e. the reflected rays are in antiphase and therefore tend to cancel. A practical criterion to delineate between rough and smooth is to define a rough surface as one for which $\Delta\theta \geqslant \pi/2$. Substituting this value into eqn. (2.26) yields the expression that for a rough surface

$$d_R \geqslant \frac{\lambda}{8\sin\psi} \tag{2.27}$$

In the mobile radio situation ψ is always very small and it is admissible to make the substitution $\sin\psi = \psi$. In these conditions eqn. (2.27) reduces to

$$d_R \geqslant \frac{\lambda}{8\psi} \tag{2.28}$$

In practice, the surface of the earth is more like that of Fig. 2.6(a) than the idealised surface that has been used to illustrate the point. The concept of a height d is therefore capable of further interpretation and in practice the value often used as a measure of terrain undulation height is σ, the standard deviation of the surface irregularities relative to the mean height.

The Rayleigh criterion is then expressed by writing eqn. (2.26) as,

$$C=\frac{4\pi\sigma\sin\psi}{\lambda}\simeq\frac{4\pi\sigma\psi}{\lambda} \tag{2.29}$$

For $C<0.1$ there is a specular reflection and the surface can be considered smooth. For $C>10$ there is highly diffuse reflection and the reflected wave is small enough to be neglected. At 900 MHz the value of σ necessary to make a surface rough, for $\psi=1°$ is about 15 m.

2.5 THE EFFECT OF THE ATMOSPHERE

The lower part of the atmosphere, known as the troposphere, is a region in which the temperature tends to decrease with height. It is separated from the stratosphere, where the air temperature tends to remain constant with height, by a boundary known as the tropopause. In general terms the height of the tropopause varies from about 9 km at the earth's poles to about 17 km at the equator. The height of the tropopause, however, also varies with atmospheric conditions, for instance at middle latitudes it may reach about 13 km in anticyclones and decline to less than about 7 km in depressions.

At frequencies above 30 MHz there are three effects worthy of mention. First, localised refractive index fluctuations can cause scattering and secondly any abrupt changes in refractive index as a function of height can cause reflection. Finally a more complicated phenomenon known as ducting can also occur; this will be discussed in the next section. All these mechanisms can carry energy beyond the normal optical horizon and therefore have the potential to cause interference between different radio communication systems. Forward scattering of radio energy is sufficiently dependable that it may be used as a mechanism for long-distance communications, especially at frequencies between about 300 MHz and 10 GHz. Nevertheless troposcatter, as it is known, is not a mechanism that is used for mobile radio communications and will not be given further consideration here. Reflection and ducting are much less predictable.

Variations in the climatic conditions within the troposphere i.e. changes of temperature, pressure and humidity cause changes in the refractive index of the air. Large scale changes of refractive index with height cause radiowaves to be refracted and the effect can be quite significant at all frequencies, at low elevation angles, especially in extending the radio horizon distance beyond that of the optical horizon. Of all the influences that the atmosphere can exert on radio signals, refraction is the one that has the greatest effect on VHF and UHF point-to-point systems and accordingly it is worthy of further discussion. We start by considering an idealised model of the atmosphere and then discuss the effects of departures from that ideal.

An ideal atmosphere is one in which the dielectric constant is unity and there is zero absorption. In practice, however, the dielectric constant of air is greater than unity and depends on the pressure and temperature of the

air and the water vapour. The dielectric constant therefore varies with weather conditions and with height above the ground. Normally, but not always, it decreases with increasing height. The consequence of changes in atmospheric dielectric constant with height is that electromagnetic waves are bent as they propagate, in a curved path that keeps them nearer to the earth than would be the case if they truly travelled in straight lines. Indeed, in so far as atmospheric influences are concerned, radiowaves behave very much like light.

The refractive index of the atmosphere at sea level differs from unity by about 300 parts in 10^6, and falls approximately exponentially with height. It is convenient to refer to the refractivity in N units, where

$$N=(n-1)10^6$$

and n is the refractive index of the atmosphere expressed as

$$n\simeq(1+300\times10^{-6})$$

A commonly-used expression for N is [1, Chap. 4]

$$N=\frac{77.6}{T}\left[P+\frac{4810e}{T}\right] \tag{2.30}$$

where P = total pressure in mb
e = water-vapour pressure in mb
T = absolute temperature

and as an example, if $P=1000$ mb, $e=10$ mb and $T=290°$ then $N=312$.

In practice, P, e and T all tend to fall exponentially with height and therefore, so does N. The value of N at a height h can therefore be written in terms of the value N_s at the earth's surface as

$$N(h)=N_s\exp(-h/H) \tag{2.31}$$

where H is a scale height (often taken as 7 km). Over the first kilometre or so, the exponential curve can be approximated by a straight line and in this region the refractivity falls by about 39 N-units. Although this may appear to be a small change, it has a profound effect on radio propagation.

In a so-called standard exponential atmosphere, i.e. one in which eqn. (2.31) applies, the refractivity decreases continuously with height and ray paths from the transmitter are therefore curved. It can be shown that the radius of curvature is given by

$$\rho=-\frac{\mathrm{dh}}{\mathrm{dn}}$$

and that in a standard atmosphere $\rho=10^6/39=25{,}640$ km. This ray path is curved and so, of course, is the surface of the earth. The geometry is illustrated in Fig. 2.7, from which it can be seen that a ray launched parallel to the earth's surface is bent downwards, but not enough to reach the ground. The distance d, from an antenna of height h to the optical horizon can be obtained

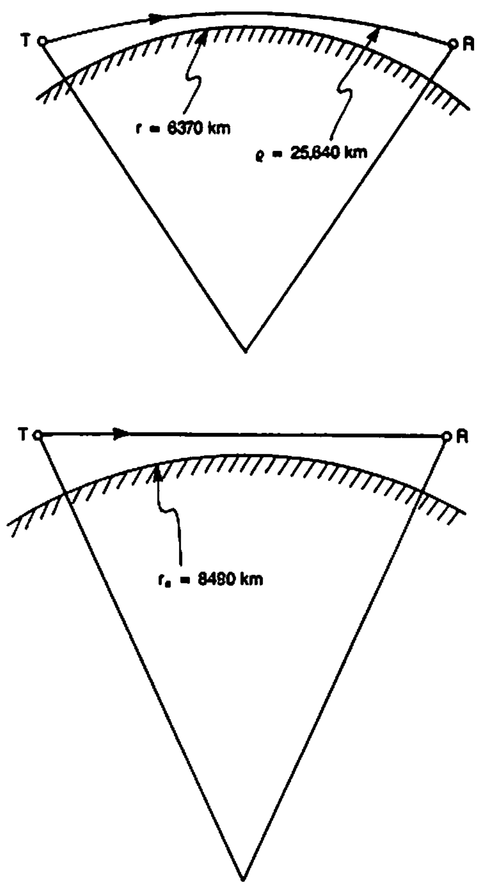

Fig. 2.7 Illustrating how the use of an effective earth radius of 8490 km (6370 × 4/3) *permits the use of straight-line propagation paths*

from the geometry of Fig. 2.1. The maximum line-of-sight range d is given by

$$d^2 = (h+r)^2 - r^2 = h^2 + 2hr \simeq 2hr \tag{2.32}$$

so that $d \simeq \sqrt{2hr}$ when $h \ll r$.

The geometry of a curved ray propagating over a curved surface is complicated and in practical calculations it is common to remove the relative curvature by increasing the true value of the earth's radius until ray paths, modified by the refractive index gradient, become straight again. The

modified radius can be found from the relationship

$$\frac{1}{r_e} = \frac{1}{r} + \frac{dn}{dh} \tag{2.33}$$

where dn/dh is the rate of change of refractive index with height.

The ratio r_e/r is the effective earth radius factor k so that the distance to the radio horizon is $\sqrt{2krh}(=\sqrt{2r_e h})$. The average value for k based on a standard atmosphere is 4/3 and use of this 'four-thirds' earth radius is very widespread in the calculation of radio paths. It leads to a very simple relationship for the horizon distance given by $d = \sqrt{2h}$ where d is in miles and h is in feet.

In practice the atmosphere does not always behave according to this idealised model and accordingly, the radio wave propagation paths are perturbed.

2.5.1 Atmospheric ducting and non-standard refraction

In a real atmosphere the refractive index may not fall continuously with height as predicted by eqn. (2.31) for a standard exponential atmosphere. There may be a general decrease, but there may also be quite rapid variations about the general trend. The relative curvature between the surface of the earth and a ray path is given by eqn. (2.33) and if dn/d$h = -1/r_e$ we have the interesting situation of zero relative curvature, i.e. a ray launched parallel to the earth's surface remains parallel to it and there is no radio horizon. The value of dn/dh necessary to cause this is -157 N-units per km ($1/6370 = 157 \times 10^{-6}$). In certain parts of the world it often turns out that the index of refraction will have a rate of decrease with height over a short distance that is greater than this critical rate which is sufficient to cause the rays to be refracted back to the surface of the earth. These rays are then reflected and refracted back again in such a manner that the field is trapped or guided in a thin layer of the atmosphere close to the earth's surface, as shown in Fig. 2.8. This is the phenomenon known as trapping or ducting. The radiowaves will then propagate over quite long distances with much less attenuation than for free space propagation because of the guiding action which is in some ways similar to that in the earth/ionosphere waveguide at low frequencies.

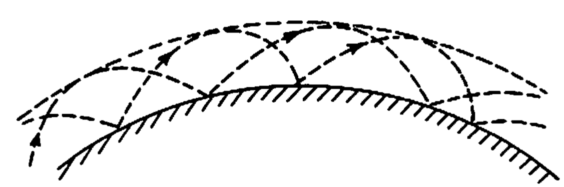

Fig. 2.8 Illustrating the phenomenon of ducting

Ducts can form near the surface of the earth (surface ducts) or at heights up to about 1500 m above the surface (elevated ducts). In order to obtain long-distance propagation both the transmitting and the receiving antennas must be located within the duct in order to couple effectively to the field in the duct. The thickness of the duct may range from a few metres to several hundred metres. In order to obtain trapping or ducting, the rays must propagate in a nearly horizontal direction and thus in order to satisfy conditions for guiding within the duct the wavelength has to be relatively small. The maximum wavelength that can be trapped in a duct of 100 m thickness is about 1 metre, (i.e. a frequency of about 300 MHz) so that the most favourable conditions for ducting are in the VHF and UHF bands. The relationship between the maximum wavelength λ and the duct thickness t, which causes good propagation is $t = 500\lambda^{2/3}$.

A simplified theory of propagation which explains the phenomenon of ducting can be expressed in terms of a modified index of refraction which is the difference between the actual refractive index and the value of $-157\,N$ units per km that causes rays to remain at a constant height above the curved surface of the earth [4, Chap 6]. Under non-standard refractive conditions, the refractive index may change either more rapidly or less rapidly than $-157\,N$ units per km. When the decrease is more rapid, the radius of curvature of ray paths is less than 25,640 km so waves propagate further without getting too far above the earth's surface. This is termed super refraction. On the other hand, when the refractive index decreases less rapidly there is less downward curvature, and substandard refraction is said to exist.

Figure 2.9 shows how changes in refractive index cause a surface duct to form and indicates some typical ray paths within the duct. Near the ground, dn/dh is negative with a magnitude greater than 157 N-units per km. At the height h_0 the gradient changes so that above this height the magnitude is less than 157. Below the height h_0 the radius of curvature of rays launched at small elevation angles is less than the radius of curvature of the earth, and above h_0 it is greater. Rays 1, 2 and 3 are trapped between the earth and an imaginary sphere at a height h_0. Rays 2 and 3 are tangential to the

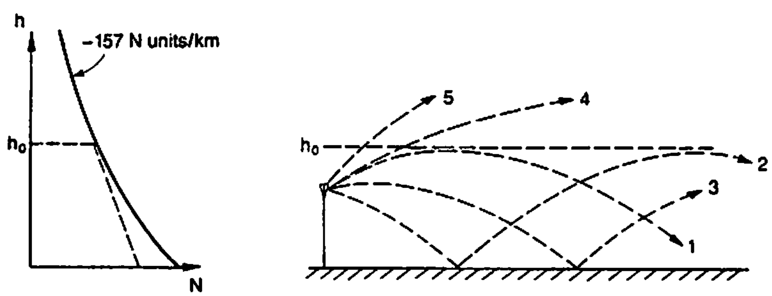

Fig. 2.9 Refractive index variation and subsequent ray paths in a surface duct

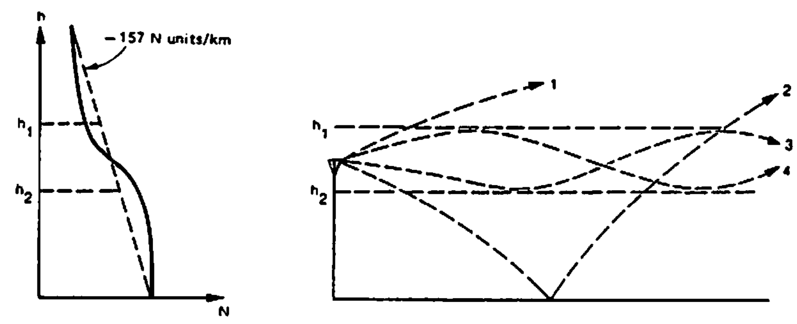

Fig. 2.10 Refractive index variation and subsequent ray paths in an elevated duct.

sphere and represent the extremes of the trapped waves. Rays 4 and 5, at high angles, are only weakly affected by the duct and resume a normal path on exit. This kind of duct can cause anomalous propagation conditions as a result of which, interference between radio services can be very severe.

Elevated ducts can also be formed, as shown in fig. 2.10. An inversion (i.e. an increasing refractive index) exists up to a height h_0 after which there is a fast decrease up to another height h_1. Rays lauched over quite a wide range of angles can become trapped in this elevated duct, the mechanism of propagation being similar to that in a surface (or ground-based) duct.

The formation of ducts is caused primarily by the water vapour content of the atmosphere since this has a stronger influence on the index of refraction than does temperature gradient. For this reason ducts commonly form over large bodies of water and in the trade-wind belt over warm seas there is often more-or-less permanent ducting, the thickness of the ducts being about 1.5 to 2 m. A quiet atmosphere is essential for ducting, hence the occurrence of ducts is a maximum in calm weather conditions over water or plains; there is too much turbulence over mountains. Ground ducts are produced by

(1) A mass of warm air arriving over a cold ground or the sea
(2) Night frosts which cause ducts during the second half of the night. (Such conditions frequently occur in desert and tropical climates.)
(3) High humidity in the lower troposphere.

Elevated ducts are caused principally by the subsidence of an air mass in a high-pressure area. As the air descends it is compressed and is thus warmed and dried. Elevated ducts occur mainly above the clouds and can interfere with ground-aircraft communications.

Anomalous propagation due to ducting can often cause television transmissions from one country to be received several hundred miles away in another country when atmospheric conditions are suitable. However ducting is not a major source of problems to mobile radio systems in temperate climates.