Homework 5 Solution

Homework Problems

1. An $(n, 1)$ repetition code takes each bit and repeats it n times. What is the minimum distance $d_{\min}$ of such a code?

   Answer: Consider, for example $n = 3$. The two codewords are 000 and 111. It is clear that $d_{\min} = 3$ in this case. In general, we have $d_{\min} = n$.

2. Suppose you have a codebook with minimum distance $d_{\min} = 5$. What is the maximum number of errors this code can correct?

   Answer: The maximum number of correctable errors is $t = \left\lfloor \frac{d_{\min}-1}{2} \right\rfloor = 2$ in this case.

3. Which encoding scheme is better, a Hamming (7,4) code or a (3,1) repetition code? Explain why.

   Answer: Both the Hamming (7,4) code and the (3,1) repetition code have a $d_{\min}$ equal to 3. So they can both correct for single bit errors. The Hamming code is better, however, because it has less redundancy. For the Hamming code, the redundancy is $\frac{7-4}{4} = 0.75$ whereas for the repetition code the redundancy is $\frac{3-1}{1} = 2$.

4. For the Hamming (7,4) code, what does the bit sequence 0101 get encoded as? Show your work. You can use a computer to check your answer but make sure you understand the procedure.

   Answer: 1100101.

5. Consider the (5,2) code

Data Bits	Codeword
00	00000
01	00001
10	10000
11	10001

   Is this a good code? Explain why or why not.

   Answer: This is not a good code. It has high redundancy $\frac{5-2}{2} = 1.5$ and it has $d_{\min} = 1$, hence it can not even correct a single bit error. A much better (5,2) code with $d_{\min} = 3$ is given in Example 6.9 of the Stallings textbook.

6. Suppose you are using a (7,4) Hamming code and receive the codeword 0101001. Confirm that this is not a valid (7,4) Hamming codeword. Determine the closest valid (7,4) Hamming codeword and identify the incorrect bit in the received codeword.

   Answer: This codeword fails the parity checks. The closest valid codeword 0111001 which has Hamming distance one from the invalid codeword. The incorrect bit is 01$\underline{0}$1001.

7. Stallings Problem 6.16.

   Answer:

   a)

$$d(00000, 10101) = 3$$
$$d(00000, 01010) = 2$$
$$d(10101, 01010) = 5.$$

   b)

$$d(000000, 010101) = 3$$
$$d(000000, 101010) = 3$$
$$d(000000, 110110) = 4$$
$$d(010101, 101010) = 6$$
$$d(010101, 110110) = 3$$
$$d(101010, 110110) = 3.$$