ECE2311D10 Exam 1

Your Name: SOLUTION Your box #: ________

March 26, 2010

- Open book, open notes.
- Calculators permitted.
- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work!
- This exam is worth a total of 100 points.

problem 1	problem 2	problem 3	problem 4	problem 5	TOTAL
40 points	20 points	10 points	10 points	20 points	100 points

1. 40 points total. Suppose you have the signal $x(t)$ shown in Figure 1.

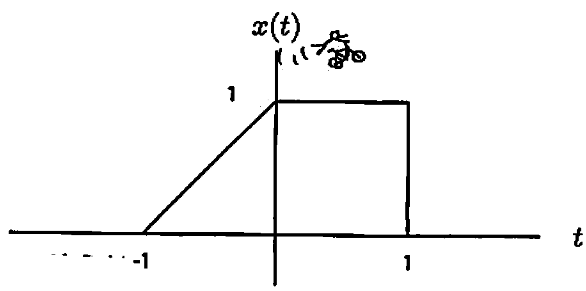
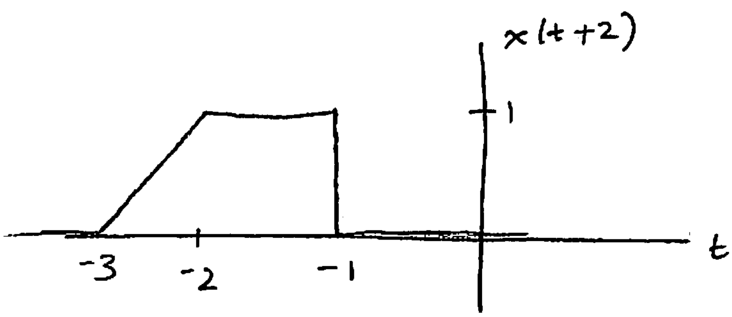
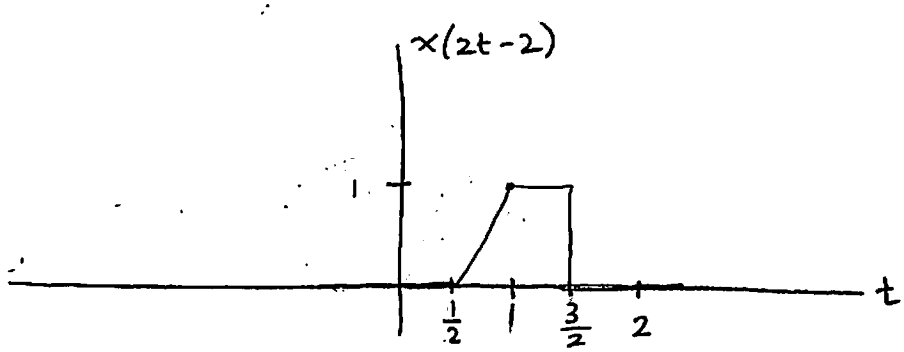


Figure 1: A signal.

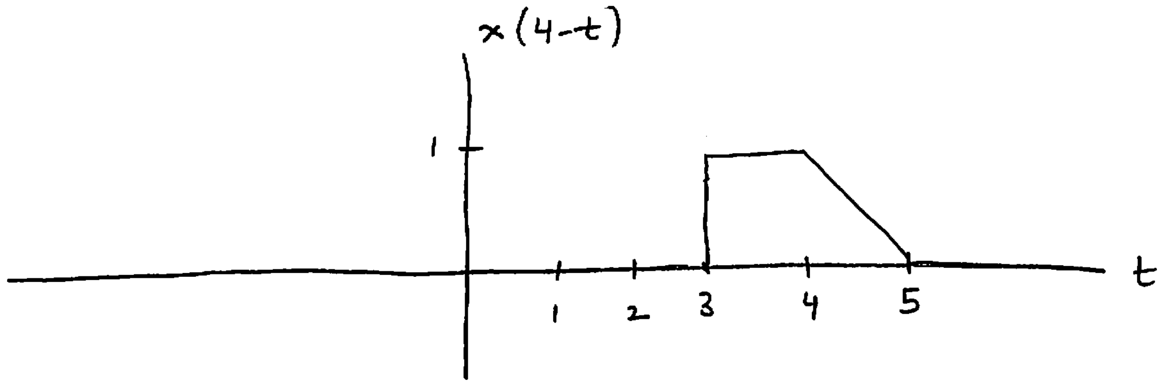
(a) 10 points. Accurately sketch $x(t+2)$.



(b) 10 points. Accurately sketch $x(2t-2)$. time shifting + time scaling

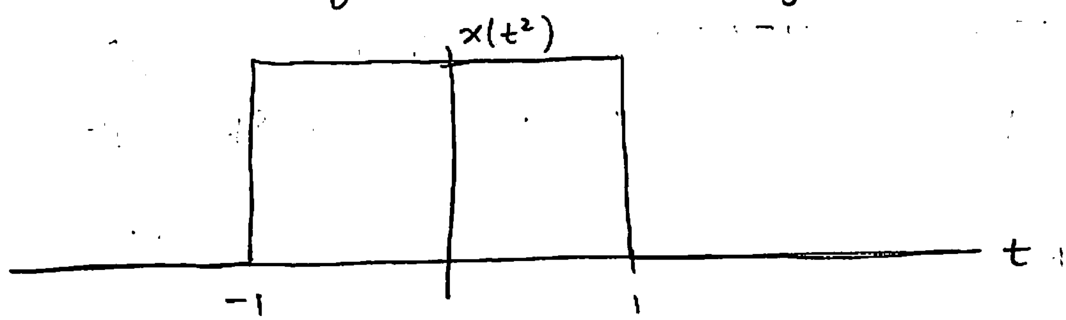


(c) 10 points. Accurately sketch $x(4-t)$. time reversal + shift



(d) 10 points. Accurately sketch $x(t^2)$.

This one requires a bit more thought.



The reason for this shape is that

$$x(t^2) = \begin{cases} 0 & t < -1 \\ 1 & -1 \leq t \leq 1 \\ 0 & t > 1 \end{cases}$$

in this range t^2 is a number between 0 and 1. $x(t) = 1$ for all t between 0 and 1.

2. 20 points. Suppose you have a system that, for a given input $x(t)$, produces the output

$$y(t) = x(t/2) + 2x(t+1) + 3x(t^2).$$

Given an input function $x(t) = 2\cos(3t)\exp(-t)u(t)$, write an expression for $y(t)$.

Method of substitution

1 $\quad t_1 = t/2 \qquad t_2 = t+1 \qquad t_3 = t^2$

2. $\quad x(t_1) + 2x(t_2) + 3x(t_3)$

3.
$$2\cos(3t_1)\exp(-t_1)u(t_1)$$
$$+ 4\cos(3t_2)\exp(-t_2)u(t_2)$$
$$+ 6\cos(3t_3)\exp(-t_3)u(t_3)$$

4.
$$y(t) = 2\cos(3t/2)\exp(-t/2)u(t/2)$$
$$+ 4\cos(3(t+1))\exp(-(t+1))u(t+1)$$
$$+ 6\cos(3t^2)\exp(-t^2)u(t^2)$$

5. Simplify. Note that $u(t^2) = 1$ for all t, so we can drop that term. Otherwise, there isn't much to do here.

$(u(t) = u(t/2))$

$$y(t) = 2\cos\left(\frac{3t}{2}\right)\exp\left(\frac{-t}{2}\right)u(t)$$
$$+ 4\cos(3t+3)\exp(-t-1)u(t+1)$$
$$+ 6\cos(3t^2)\exp(-t^2).$$

3. 10 points. Suppose you have a system that, for a given input $x(t)$, produces the output

$$y(t) = x(t - 0.1)$$

Given an input $x(t) = \cos(2t)$, write the output as $y(t) = \cos(2t + \theta)$, i.e. determine θ in radians. Discuss how the signal $y(t)$ leads or lags the signal $x(t)$.

$$y(t) = \cos(2(t - 0.1)) = \cos(2t - 0.2)$$

$y(t)$ <u>lags</u> the input $x(t)$ by 0.2 radians.

4. 10 points. Suppose you have an unknown system that you are interested in characterizing. You find a function generator and apply an input $x_1(t) = \cos(\omega t)$. You measure the output with an oscilloscope to be $y_1(t) = \cos(2\omega t)$. You then try another input, $x_2(t) = \cos(2\omega t)$ and get an output $y_2(t) = \cos(4\omega t)$.

Do you have enough information to determine whether this is a linear system or not? If you do have enough information, is this system is linear or not?

This system can't be linear because linear systems only change the amplitude and/or phase of sinusoidal inputs. This system is doubling the input frequency, hence it can't be linear.

The second experiment was unnecessary.

5. 20 points total. Characterize the following systems according to linear/nonlinear, time-invariant/time-varying, dynamic/instantaneous, and causal/non-causal.

(a) 10 points. $y(t) = 1 - x(t+1)$.

Nonlinear. Suppose $x(t) = u(t)$, then $y_1(t) = 1 - u(t+1)$
Now suppose $x_2(t) = 2u(t)$, Then $y_2(t) = 1 - 2u(t+1)$
but $y_2(t) \neq 2y_1(t)$, so nonlinear.

Noncausal. output at time t depends on future inputs

Time invariant: If we apply $x(t-\tau)$, we get
$z(t) = 1 - x(t-\tau+1) = y(t-\tau)$ ✓.

dynamic: output does not depend on current input, but future inputs. The system needs memory

(b) 10 points. $y(t) = \frac{d^2}{dt^2}x(t-1)$

Linear. $y_1(t) = \frac{d^2}{dt^2} x_1(t-1)$
$y_2(t) = \frac{d^2}{dt^2} x_2(t-1)$

Apply $a x_1(t) + b x_2(t)$ to get an output of
$y(t) = \frac{d^2}{dt^2}\left(a x_1(t-1) + b x_2(t-1)\right) = a \frac{d^2}{dt^2} x_1(t-1) + b \frac{d^2}{dt^2} x_2(t-2)$
$= a y_1(t) + b y_2(t)$. ✓

Causal: output only depends on past inputs

time invariant: If we apply $x(t-\tau)$, we get
$z(t) = \frac{d^2}{dt^2} x(t-\tau) = y(t-\tau)$

dynamic: This system needs memory to keep past inputs and to compute the derivative.