ECE2311D10 Exam 2

Your Name: SOLUTION Your box #: ____________

April 2, 2010

- Open book, open notes.
- Calculators permitted.
- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work!
- This exam is worth a total of 100 points.

problem 1	problem 2	problem 3	problem 4	TOTAL
20 points	20 points	30 points	30 points	100 points

1. 20 points. Suppose you have the periodic signal $x(t)$ given in Figure 1 below. Compute the power and energy of this signal.

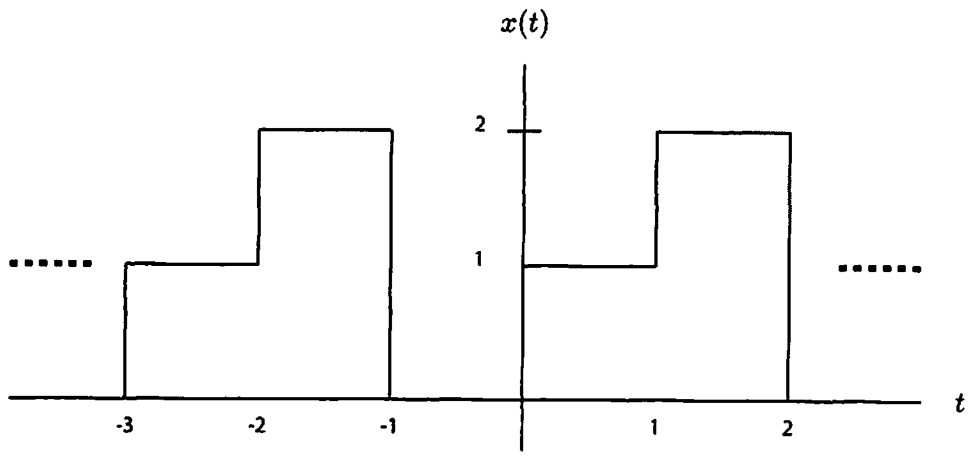


Figure 1: A signal.

The energy of this signal is <u>infinite</u> because

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt \quad \text{is unbounded.}$$

To compute the power, we can look at just one period. $T_0 = 3$, so

$$P_x = \frac{1}{3}\int_{-1}^{2} x^2(t)\,dt = \frac{1}{3}\left[1+4\right] = 5/3$$

hence $\boxed{P_x = \frac{5}{3}}$

2. 20 points. Without using any math, discuss the useful things you can do with convolution.

Convolution allows you to compute the output of a linear time invariant system S with impulse response $h(t)$ for any input $x(t)$.

Hence, the utility of convolution is that, if you can determine the impulse response of a system by analysis or experiment, you can analyze the behavior of the system for any arbitrary input. You do not need to do a new experiment or re-solve your differential equations ~~for~~ different inputs. You just need to compute the convolution integral.

3. 30 points. Compute the output $y(t) = x(t) * h(t)$ of a linear time invariant system with impulse response

$$h(t) = u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

given the input $x(t)$ shown in Figure 2.

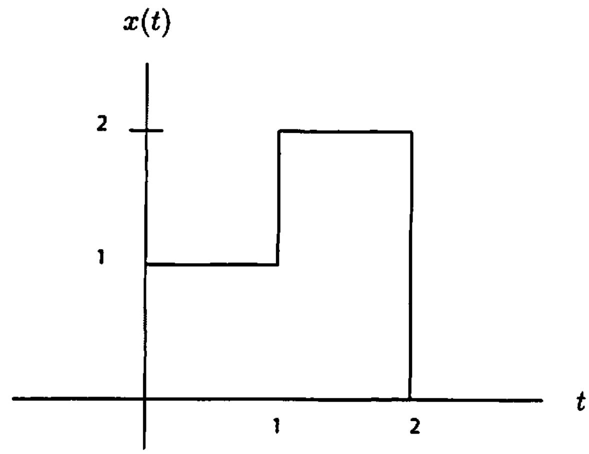
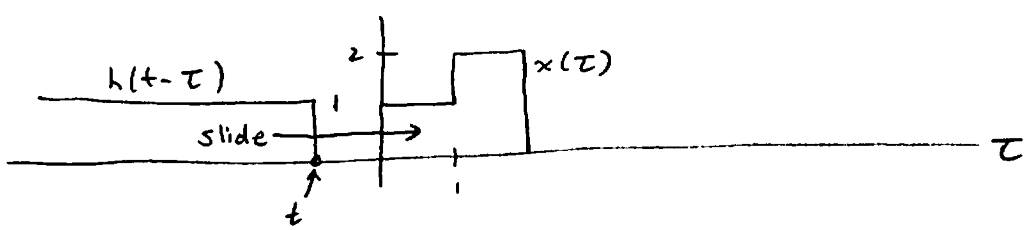


Figure 2: Input signal.

Graphical analysis:

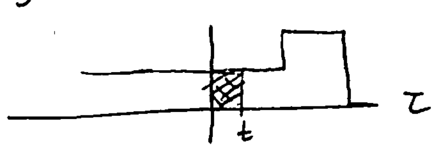


4 cases:

1. No overlap $(t<0)$
2. Partial overlap with first step $(0 \leq t < 1)$
3. Partial overlap with second step $(1 \leq t < 2)$
4. Full overlap $(t \geq 2)$

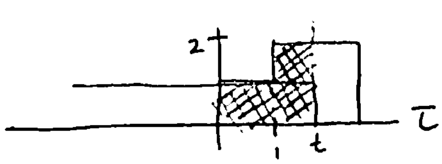
case 1 : $y(t) = 0$

case 2 :



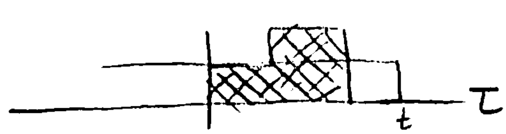
$y(t) = t$

case 3 :



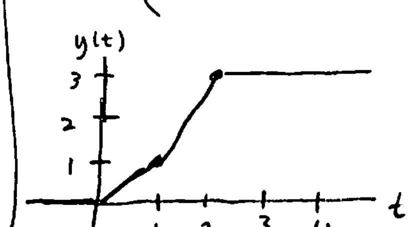
$y(t) = 1 + 2(t-1)$

case 4 :



$y(t) = 3$

$$y(t) = \begin{cases} 0 & t<0 \\ t & 0 \leq t < 1 \\ 2t-1 & 1 \leq t < 2 \\ 3 & t \geq 2 \end{cases}$$



4. 30 points. Compute $y(t) = x(t) * h(t)$ for $x(t) = (t-1)u(t-1)$ and $h(t) = e^{-t}u(t)$.

you can do this straight from the table, accounting for the timeshift in $x(t)$.

Line 7:

$$(t^N u(t)) * (e^{\lambda t} u(t)) \qquad N=1 \qquad \lambda = -1$$

$$(t\,u(t)) * (e^{-t} u(t)) = \frac{e^{-t}}{1} u(t) - \sum_{k=0}^{1} \frac{1 \cdot t^{1-k}}{(-1)^{k+1}(1-k)!} u(t)$$

$$= \left[e^{-t} + t - 1 \right] u(t)$$

Now apply the delay:

$$y(t) = \left[e^{-(t-1)} + (t-1) - 1 \right] u(t-1)$$

simplify

$$\boxed{y(t) = \left[e^{-(t-1)} + t - 2 \right] u(t-1)}$$

5. 20 point BONUS. Using your result from the previous problem, describe how you could compute $y(t) = x(t) * h(t)$ for $x(t) = tu(t-1)$ and $h(t) = e^{-t}u(t)$.

$$x(t) = t\,u(t-1) = (t-1)u(t-1) + u(t-1)$$

☆ Distributive property ☆

$$y(t) = x(t) * h(t)$$

$$= \underbrace{\big((t-1)u(t-1)\big) * \left(e^{-t}u(t)\right)}_{\text{we already have this from the previous problem}} + \underbrace{\left(u(t-1)\right) * \left(e^{-t}u(t)\right)}_{\text{we have this from line 2 of the table, also accounting for the time shift}}$$

$$\left(u(t)\right) * \left(e^{-t}u(t)\right) = \left(1 - e^{-t}\right)u(t)$$

so

$$\left(u(t-1)\right) * \left(e^{-t}u(t)\right) = \left(1 - e^{-(t-1)}\right)u(t-1)$$

so

$$y(t) = \Big[\overbrace{e^{-(t-1)} + t - 2}\; + 1 - e^{-(t-1)}\Big]u(t-1)$$

$$\boxed{y(t) = (t-1)u(t-1)}$$