ECE2311D10 Exam 5

Your Name: SOLUTION Your box #: ____________

April 23, 2010

- Open book, open notes.
- Calculators permitted.
- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work/reasoning!
- This exam is worth a total of 100 points.

problem 1	problem 2	problem 3	TOTAL
30 points	30 points	40 points	100 points

1. 30 points. Find the Fourier transform of the aperiodic signal $x(t)$ shown in Figure 1 below. Note that

$$\frac{d}{dt}x(t) = \frac{1}{2}\text{rect}\left(\frac{t}{4}\right) - 2\delta(t).$$

You can compute the answer to this problem without integration by using the Fourier transform table and properties in your book. **Simplify your answer as much as possible..**

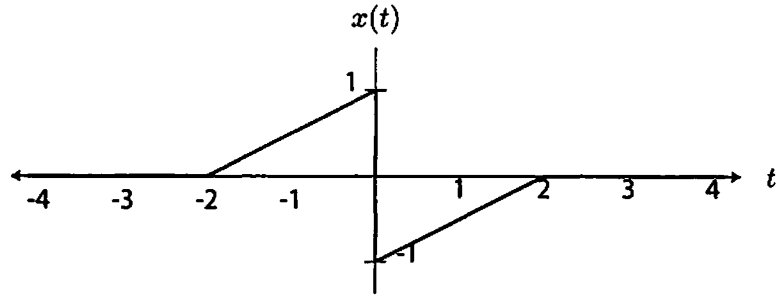


Figure 1: An aperiodic signal.

let $x_1(t) = \frac{1}{2}\,\text{rect}\left(\frac{t}{4}\right) - 2\delta(t)$

then, from the table on p. 702,

$$X_1(\omega) = \frac{1}{2}\cdot 4\,\text{sinc}\left(\frac{\omega \cdot 4}{2}\right) - 2$$

$$X_1(\omega) = 2\,\text{sinc}(2\omega) - 2$$

since $x_1(t) = \frac{d}{dt}x(t)$

$$X_1(\omega) = j\omega X(\omega) \qquad \text{(time differentiation property)}$$

hence

$$\boxed{X(\omega) = \frac{2}{j\omega}\left[\text{sinc}(2\omega) - 1\right] = \frac{2j}{\omega}\left[1 - \text{sinc}(2\omega)\right]}$$

2. 30 points. Find the inverse Fourier transform of $X(\omega)$ shown in Figure 2 below. **Simplify your answer as much as possible using Euler's identity and other trigonometric identities.** Hint: You can save some time here by using one or more properties of the Fourier transform.

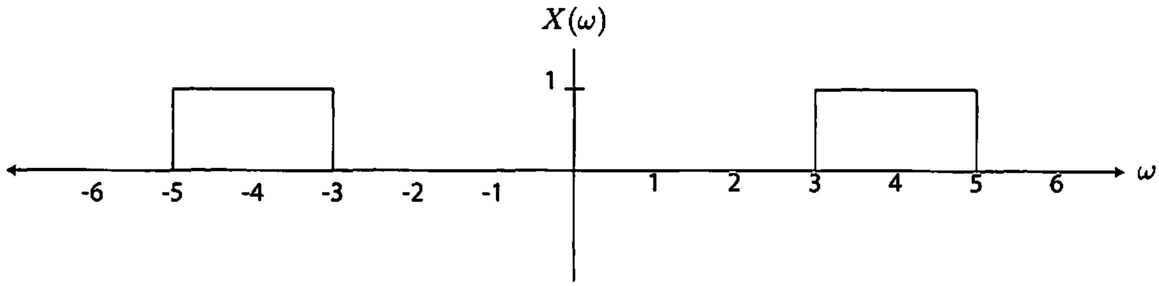
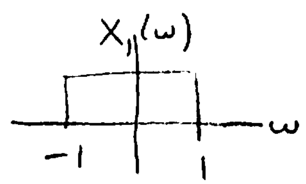


Figure 2: Frequency domain representation. Note that $\angle X(\omega) = 0$ for all ω.

Line 18 of table : $\frac{W}{\pi} \operatorname{sinc}(Wt) \longleftrightarrow \operatorname{rect}\left(\frac{\omega}{2W}\right)$



this is $\operatorname{rect}\left(\frac{\omega}{2}\right)$ hence $W=1$

Hence $x_1(t) = \frac{1}{\pi} \operatorname{sinc}(t)$

Now use the frequency shifting property...

$$x(t) = \frac{1}{\pi} \operatorname{sinc}(t)\, e^{j4t} + \frac{1}{\pi} \operatorname{sinc}(t)\, e^{-j4t}$$

$$= \frac{1}{\pi} \operatorname{sinc}(t) \left[e^{j4t} + e^{-j4t} \right]$$

$$= \frac{1}{\pi} \operatorname{sinc}(t)\, 2\cos(4t)$$

$$\boxed{x(t) = \frac{2}{\pi} \cos(4t) \operatorname{sinc}(t)}$$

3. 40 points total. Suppose you apply an input signal $x(t) = e^{-3(t-1)}u(t-1)$ to a linear time-invariant system with impulse response $h(t) = e^{-t}u(t)$. Compute the output of the system $y(t)$ using Fourier analysis.

a)

let $v(t) = e^{-3t} u(t)$ Then $V(\omega) = \dfrac{1}{3+j\omega}$

(from line 1 of the table)

since $x(t) = v(t-1)$ we can use the time shifting property to write

$$X(\omega) = V(\omega) e^{-j\omega} = \frac{e^{-j\omega}}{3+j\omega}$$

$H(\omega)$ can be also computed from line 1 of the table

$$H(\omega) = \frac{1}{1+j\omega}$$

Hence $Y(\omega) = \dfrac{e^{-j\omega}}{(1+j\omega)(3+j\omega)}$

b) To compute $y(t)$, we need to perform partial fraction expansion

$$e^{-j\omega}\left[\frac{K_1}{1+j\omega} + \frac{K_2}{3+j\omega}\right] = e^{-j\omega}\left[\frac{1}{(1+j\omega)(3+j\omega)}\right]$$

$$K_1(3+j\omega) + K_2(1+j\omega) = 1 + 0j\omega$$

$$\left.\begin{aligned} 3K_1 + K_2 &= 1 \\ K_1 + K_2 &= 0 \end{aligned}\right\} \Rightarrow 2K_1 = 1 \Rightarrow K_1 = \tfrac{1}{2}, \quad K_2 = -\tfrac{1}{2}$$

Hence $Y(\omega) = e^{-j\omega}\left[\dfrac{1/2}{1+j\omega} - \dfrac{1/2}{3+j\omega}\right] \Rightarrow \frac{1}{2}e^{-(t-1)}u(t) - \frac{1}{2}e^{-3(t-1)}u(t)$

this just causes a timeshift of one second.

corrected

$$\boxed{y(t) = \frac{1}{2}\left[e^{-(t-1)} - e^{-3(t-1)}\right]u(t-1)}$$