ECE2311D10 Comprehensive Final Exam

Your Name: SOLUTION Your box #: ________

May 4, 2010

- Open book, open notes.
- Calculators permitted.
- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work/reasoning!
- This exam is worth a total of 200 points.

problem 1	problem 2	problem 3	problem 4	TOTAL
50 points	50 points	50 points	50 points	200 points

1. 50 points total. Given the circuit shown in Figure 1, answer the following questions.

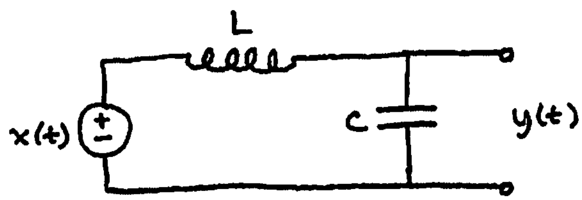


Figure 1: A circuit.

(a) 25 points. Find the transfer function $H(s)$.

resistor divider...

$$Y(s) = \frac{\frac{1}{Cs}}{Ls + \frac{1}{Cs}} X(s) = \frac{1}{LCs^2 + 1} X(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}} X(s)$$

hence

$$\boxed{H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{1}{LC}}}$$

(b) 25 points. Find the impulse response $h(t)$.

From table on p. 344, line 8b

$$\sin(bt)u(t) \longleftrightarrow \frac{b}{s^2+b^2}$$

set $b = \frac{1}{\sqrt{LC}}$ to get

$$\boxed{h(t) = \frac{1}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}}\right) u(t)}$$

2. 50 points. Suppose you have an LTI system with impulse response

$$h(t) = e^{-t}\sin(t)u(t).$$

Compute the output of this system given an input of $x(t) = \cos(3t)$.

Since this input is just a periodic signal at a fundamental frequency of $\omega_0 = 3$ (with no harmonics), we just need to compute $H(\omega)$ and evaluate the magnitude and phase at $\omega = \omega_0$.

Table p. 702, line 15

$$e^{-at}\sin(\omega_0 t)u(t) \longleftrightarrow \frac{\omega_0}{(a+j\omega)^2+\omega_0^2} \qquad a>0$$

We have $a = 1$ ✓

$\omega_0 = 1$

so

$$H(\omega) = \frac{1}{(1+j\omega)^2+1}$$

Evaluate at $\omega = \omega_0 = 3$

$$H(3) = \frac{1}{(1+3j)^2+1} = -0.0824 - 0.0706j$$

$|H(3)| = 0.1085$

$\measuredangle H(3) = -2.433$ radians.

Hence $\boxed{y(t) = 0.1085\cos(3t - 2.433)}$

3. 50 points. Suppose you have a LTI system with impulse response

$$h(t) = \frac{1}{\pi}\text{sinc}(t)$$

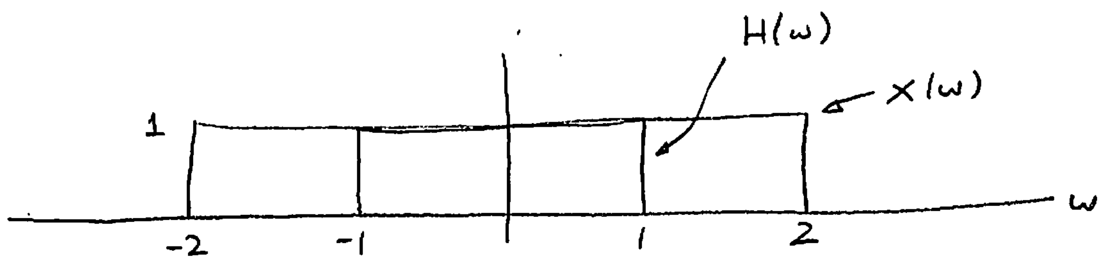
and you apply an input signal

$$x(t) = \frac{2}{\pi}\text{sinc}(2t).$$

Compute the output $y(t)$ of this system.

$$\left.\begin{array}{ll} \text{Note that} & \mathcal{F}\{h(t)\} = \text{rect}\left(\frac{w}{2}\right) = H(\omega) \\ \text{and} & \mathcal{F}\{x(t)\} = \text{rect}\left(\frac{w}{4}\right) = X(\omega) \end{array}\right\}$$ line 18 of table on p. 702

sketch ... (see rect function on p. 687)



$Y(\omega) = H(\omega)X(\omega) = H(\omega)$

hence $y(t) = h(t)$

$$\boxed{y(t) = \frac{1}{\pi}\text{sinc}(t)}$$

4. 50 points. Given $x(t)$ shown in Figure 2 below, compute $y(t) = x(t) * u(t)$.

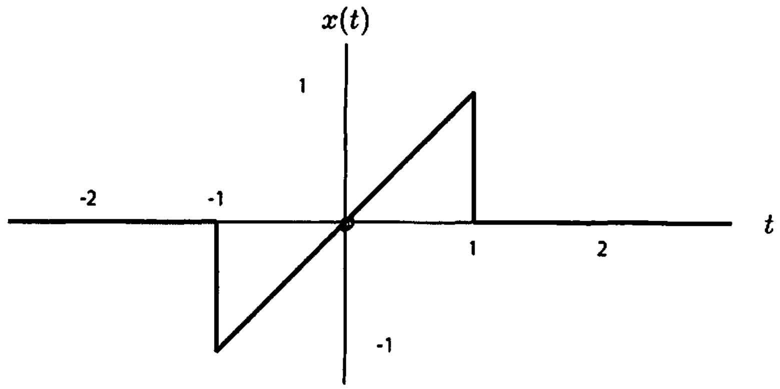
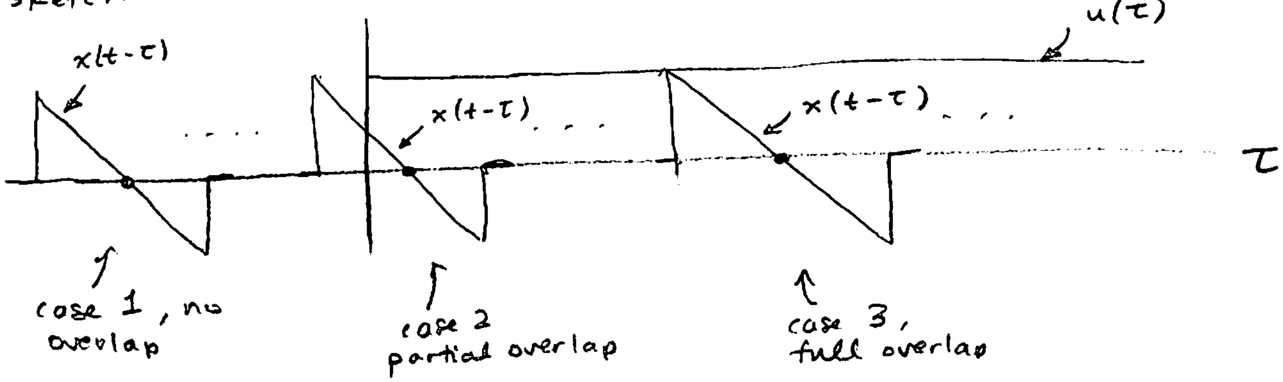


Figure 2: A signal.

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, x(t-\tau)\, d\tau$$

sketch



case 1 : $t < -1$, no overlap, $y(t) = 0$

case 2 : $-1 \le t < 1$, partial overlap

$$y(t) = \int_0^{t+1} 1 \cdot (t-\tau)\, d\tau = t(t+1) - \int_0^{t+1} \tau\, d\tau = t^2 + t - \frac{(t+1)^2}{2}$$

$$y(t) = t^2 + t - \frac{t^2 + 2t + 1}{2} = \frac{t^2 - 1}{2}$$

case 3 : $t \ge 1$, full overlap

$y(t) = 0$ because there is as much area above the axis as below the axis.

Hence
$$y(t) = \begin{cases} \frac{t^2-1}{2} & -1 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$