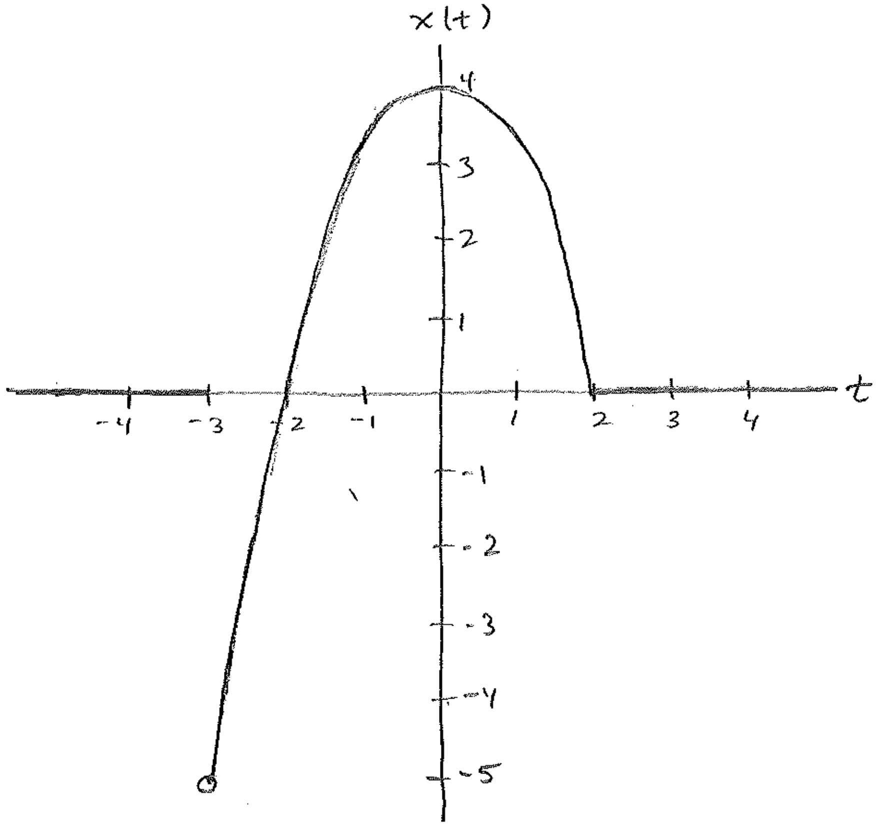
1. $x(t) = \begin{cases} 4-t^2 & -3<t<2 \\ 0 & \text{otherwise} \end{cases}$




See Matlab plot on following page

2. a) $y_1(t) = x(t+1) = \begin{cases} 4-(t+1)^2 & -3<t+1<2 \\ 0 & \text{otherwise} \end{cases}$

$= \begin{cases} 3-2t-t^2 & -4<t<1 \\ 0 & \text{otherwise} \end{cases}$

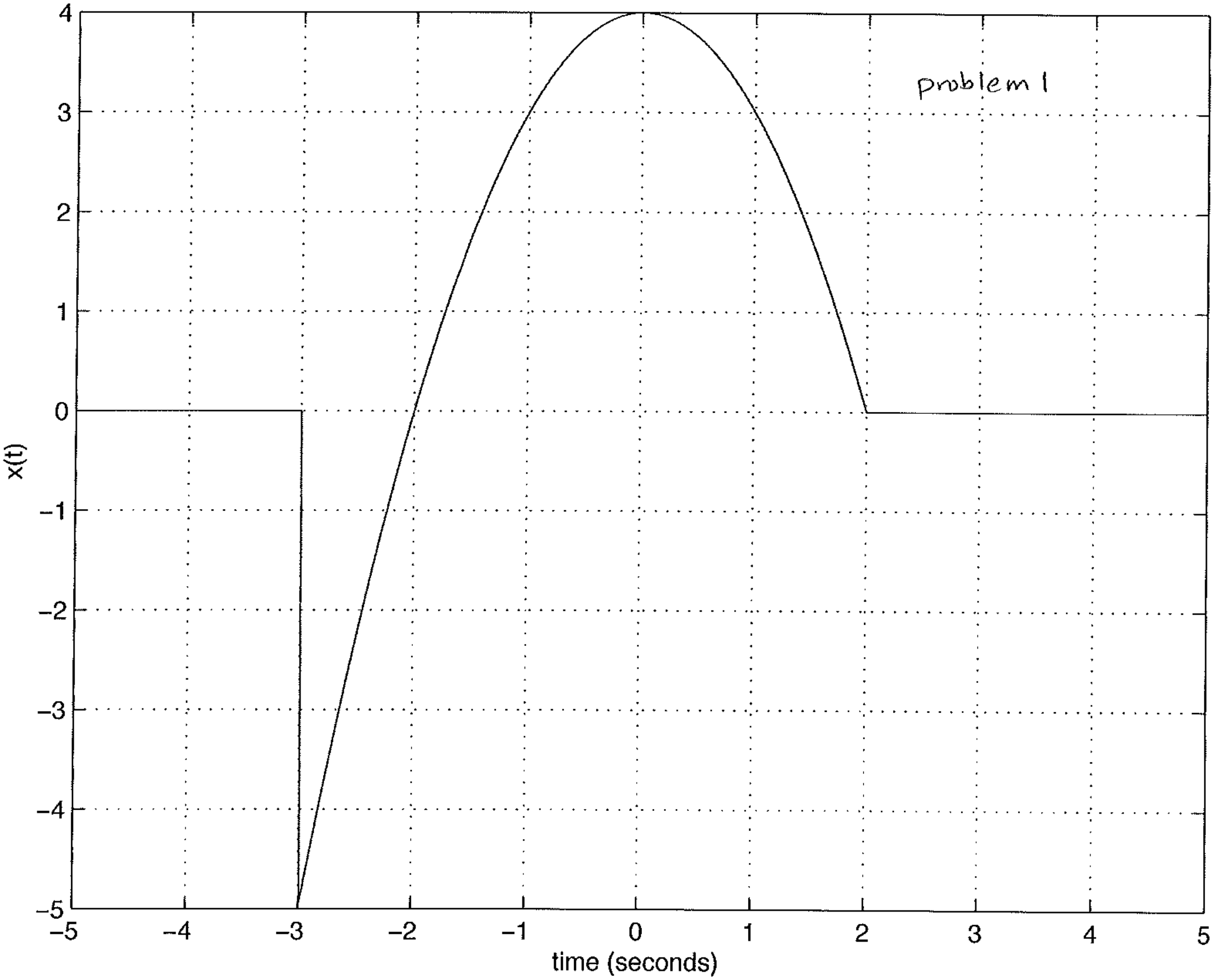
b) $y_2(t) = x(\frac{t}{2}) = \begin{cases} 4-(\frac{t}{2})^2 & -3<\frac{t}{2}<2 \\ 0 & \text{otherwise} \end{cases}$

$= \begin{cases} 4-\frac{1}{4}t^2 & -6<t<4 \\ 0 & \text{otherwise.} \end{cases}$

c) $y_3(t) = x(1-t) = \begin{cases} 4-(1-t)^2 & -3<1-t<2 \\ 0 & \text{otherwise} \end{cases}$

$= \begin{cases} 3+2t-t^2 & -1<t<4 \\ 0 & \text{otherwise} \end{cases}$

(continued on page 3)


problem 1
x(t)
time (seconds)
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5

d) $y_4(t) = x(3t+3) + 2x(t^2)$

method of substitution:

step1: let $t_1 = 3t+3$

$t_2 = t^2$

step2: $y_4(t) = x(t_1) + 2x(t_2)$

step3: $y_4(t) = \begin{cases} 4-t_1^2 & -3<t_1<2 \\ 0 & \text{otherwise} \end{cases} + 2\begin{cases} 4-t_2^2 & -3<t_2<2 \\ & \text{otherwise} \end{cases}$

step 4: $y_4(t) = \begin{cases} 4-(3t+3)^2 & -3<3t+3<2 \\ 0 & \text{otherwise} \end{cases}$

$+2\begin{cases} 4-t^4 & -3<t^2<2 \\ 0 & \text{otherwise.} \end{cases}$

note that this is the same as $t^2<2$

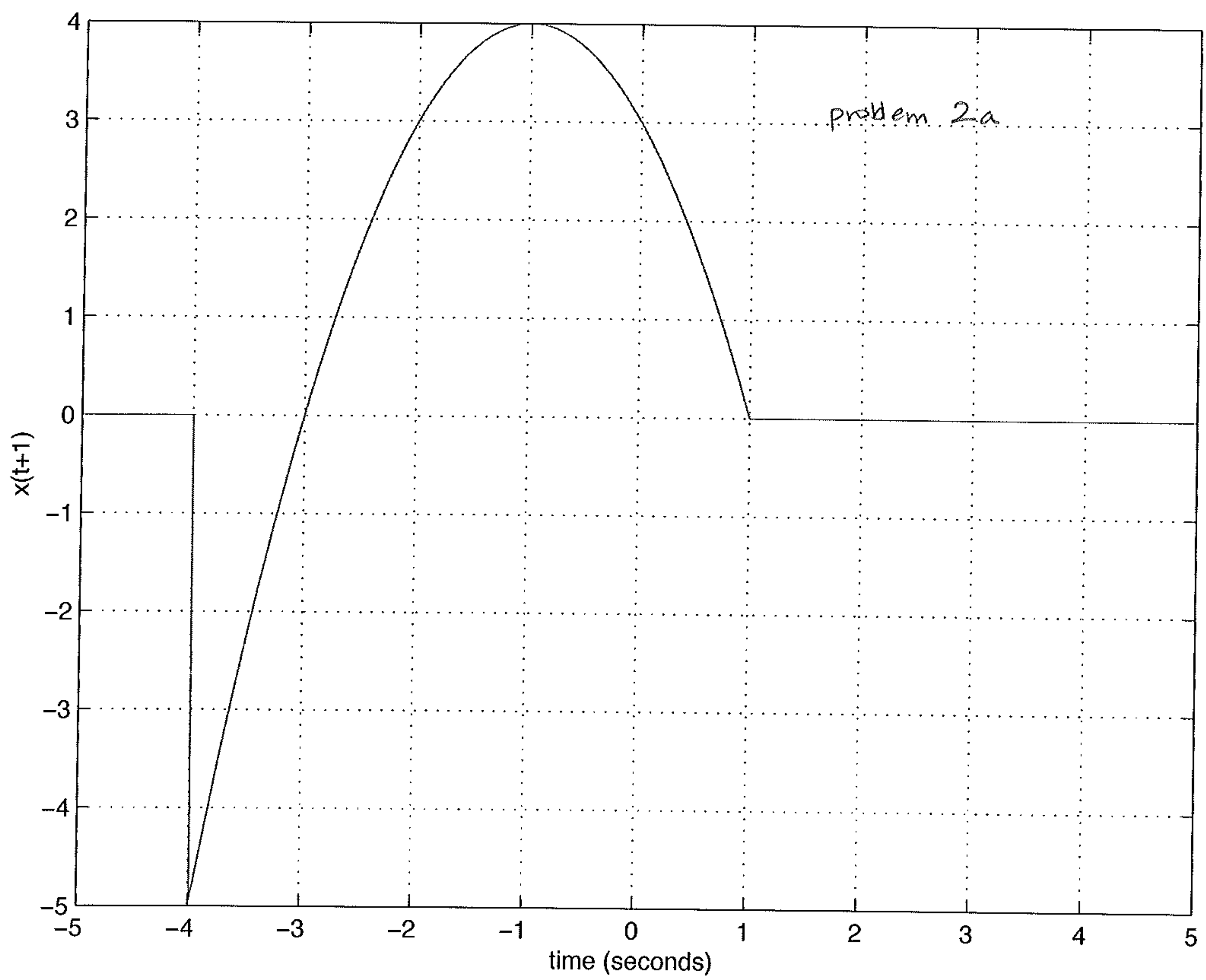
Simplify...

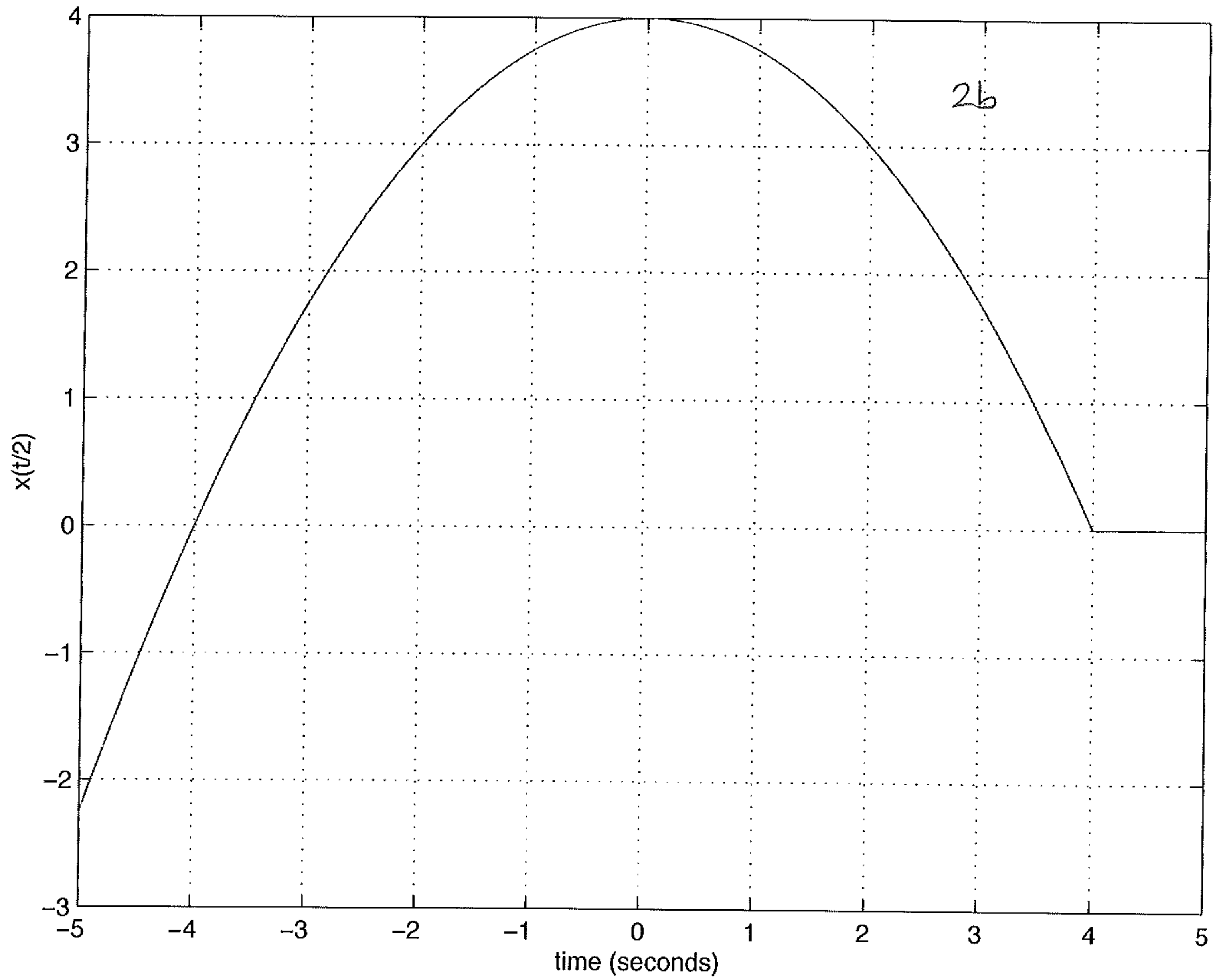
$y_4(t) = \begin{cases} -5-18t-9t^2 & -2<t<-\frac{1}{3} \\ 0 & \text{otherwise} \end{cases}$

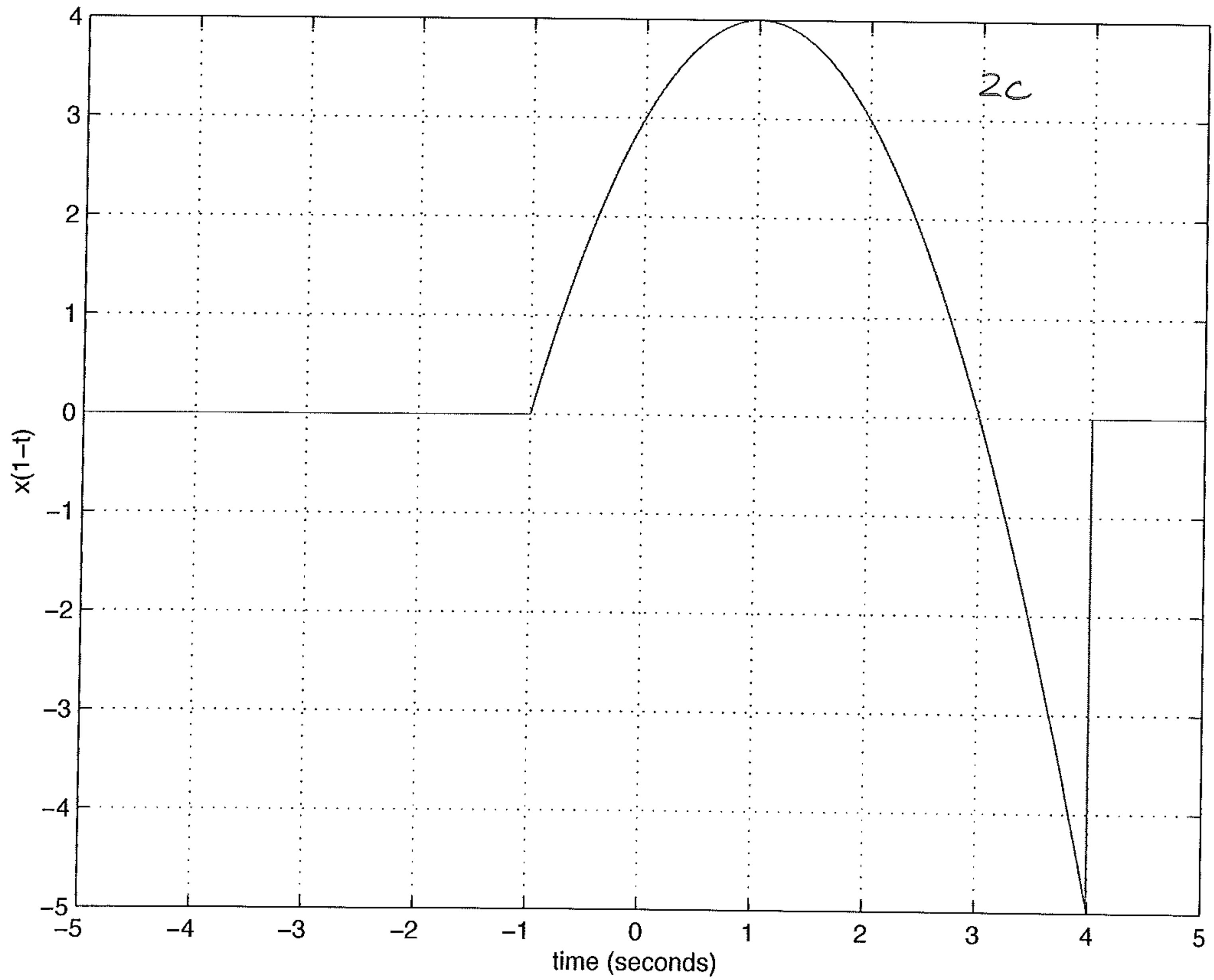
$+\begin{cases} 8-2t^4 & -\sqrt{2}<t<\sqrt{2} \\ 0 & \text{otherwise.} \end{cases}$

$y_4(t) = \begin{cases} -5-18t-9t^2 & -2<t\le-\sqrt{2} \\ 3-18t-9t^2-2t^4 & -\sqrt{2}<t<-\frac{1}{3} \\ 8-2t^4 & -\frac{1}{3}\le t<\sqrt{2} \\ 0 & \text{otherwise} \end{cases}$

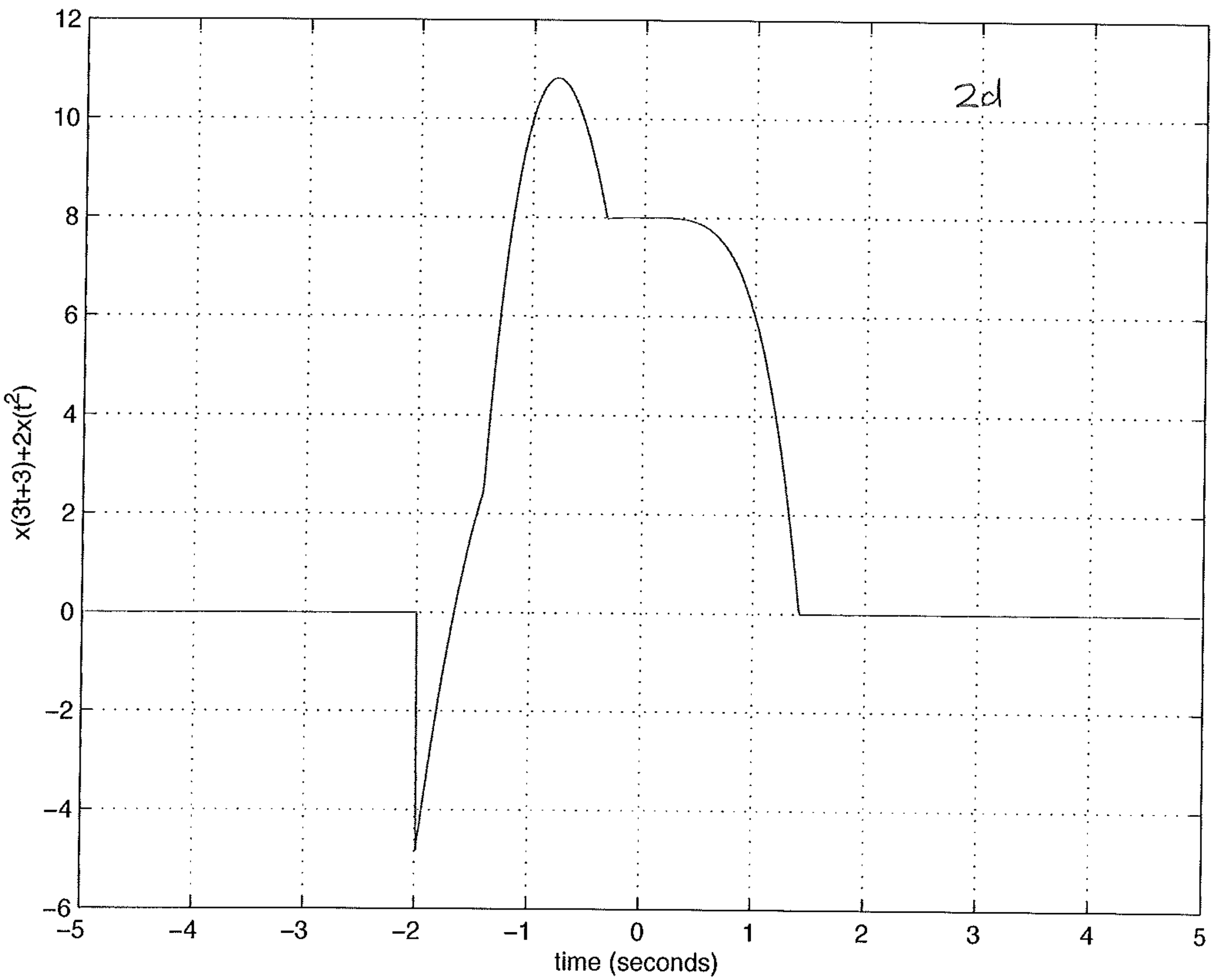
you should hand sketch each of these for practice.
Matlab plots are on the following pages.


problem 2a
x(t+1)
time (seconds)
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5


2b
x(t/2)
time (seconds)
4
3
2
1
0
−1
−2
−3
−5
−4
−3
−2
−1
0
1
2
3
4
5

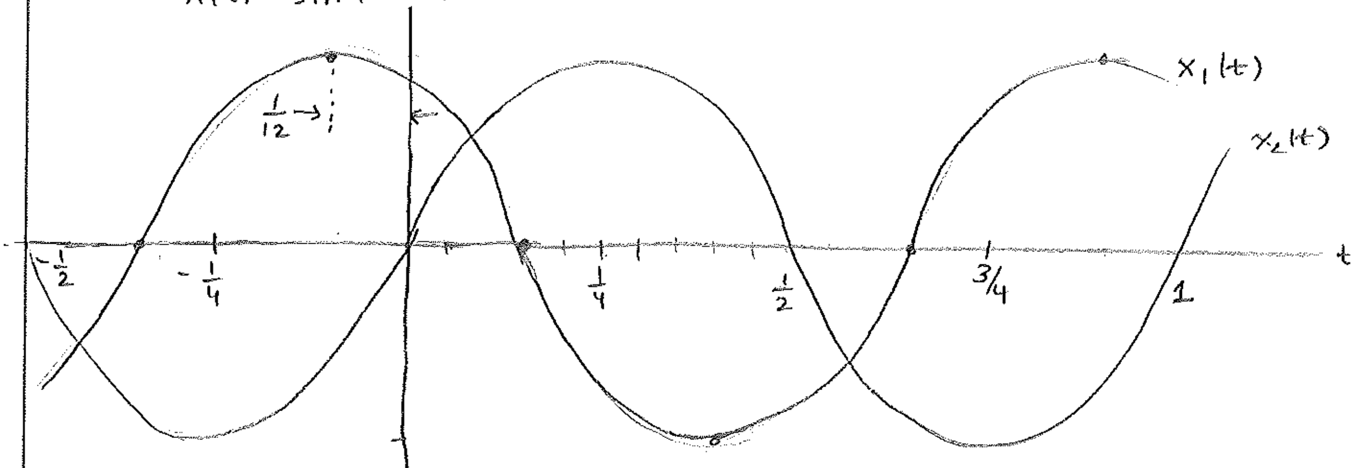


2c
x(1−t)
time (seconds)
4
3
2
1
0
−1
−2
−3
−4
−5
−5
−4
−3
−2
−1
0
1
2
3
4
5


2d
x(3t+3)+2x(t^2)
time (seconds)
12
10
8
6
4
2
0
−2
−4
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5

3. $x_1(t) = \cos(2\pi t + \pi/6) = \cos\left(2\pi\left(t + \frac{1}{12}\right)\right)$ ↑ time shift (advance)

$x(t) = \sin(2\pi t)$




Either of the following answers is ok:

$x_1(t)$ leads $x_2(t)$ by $\frac{2\pi}{3}$

$x_2(t)$ lags $x_1(t)$ by $2\pi/3$

This is also consistent with the fact that "sin lags cos by $\frac{\pi}{2}$"

$\Rightarrow x_2(t)$ lags $x_1(t)$ by $\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$ ✓

4. Fundamental period = 4 seconds

Fundamental freq = $\frac{1}{4}$ Hz $= \frac{\pi}{2}$ rad/sec

5. $y(t) = \begin{cases} t\,x(t) & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$ $\qquad x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t)$

a) Apply $a x_1(t) + b x_2(t)$ to this system

$$y(t) = \begin{cases} t\,(a x_1(t) + b x_2(t)) & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$= a \begin{cases} t\,x_1(t) & t \geq 0 \\ 0 & \text{otherwise} \end{cases} + b \begin{cases} t\,x_2(t) & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

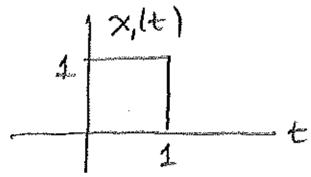
$$= a y_1(t) + b y_2(t)$$

$\Rightarrow$ system is linear

b) This is a memoryless system because the output at time $t$ only depends on the input at time $t$ and $t$ itself.

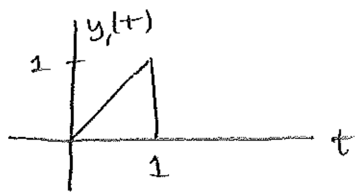
c) This is a causal system because the output at time $t$ depends only on the input at time $t$ and $t$ itself. It does not depend on any future inputs.

d) This system is time varying. To see why, suppose

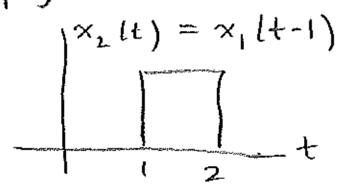



is the input.

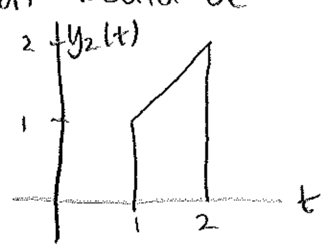
Then the output would be




Now apply the same input delayed by 1 second.




The output would be




Note that $y_2(t) \neq y_1(t-1) \Rightarrow$ time varying.

6. $y(t) = \frac{d}{dt} x(t-1)$

a) skip

b) This system is linear because the derivative is linear. But you should check it to be sure.

Apply an input of $a x_1(t) + b x_2(t)$

Then $y(t) = \frac{d}{dt}\left(a x_1(t-1) + b x_2(t-2)\right)$

$$= a \frac{d}{dt} x_1(t-1) + b \frac{d}{dt} x_2(t-2)$$

$$= a y_1(t) + b y_2(t)$$

c) This system is not memoryless since it depends on $x(t-1)$. The derivative is also not memoryless.

d) This system is causal because it does not depend on future inputs. You can see this explicitly by recalling the defn of the derivative.

$$\frac{d}{dt} x(t) = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}$$

hence $y(t) = \lim_{h \to 0} \frac{x(t-1+h) - x(t-1)}{h}$

Both $x(t-1+h)$ and $x(t-1)$ are past inputs, not future inputs.

e) To test time invariance, we apply a time shifted input and check the output to see if it has the same shift.

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow \frac{d}{dt} x(t-1) = y(t)$$

$$x(t+T) \longrightarrow \boxed{\text{system}} \longrightarrow \frac{d}{dt} x(t+T-1) = y(t+T)$$

so, yes, the system is time invariant.

TOPS.
35500