ECE2311 HW4 SOLUTION

1. Lathi 6.3-1

(b) $T_0 = 10\pi$, $\omega_0 = 2\pi/10\pi = 1/5$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t},$$

where $$D_n = \frac{1}{10\pi}\int_{-\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt = \frac{j}{2\pi n}\left(-2j\sin\frac{n\pi}{5}\right) = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

(c)

$$x(t) = D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jnt}, \quad \text{where, by inspection} \quad D_0 = 0.5$$

$$D_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi} e^{-jnt}\,dt = \frac{j}{2\pi n},$$

so that $$|D_n| = \frac{1}{2\pi n}, \quad \text{and} \quad \angle D_n = \begin{cases} \frac{\pi}{2} & n > 0 \\ \frac{-\pi}{2} & n < 0 \end{cases}$$

2. Lathi 6.3-3

(a)

$$x(t) = 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right)$$

For a compact trigonometric form, all terms must have cosine form and amplitudes must be positive. For this reason, we rewrite $x(t)$ as

$$\begin{aligned} x(t) &= 3\cos t + \cos\left(5t - \frac{\pi}{6} - \frac{\pi}{2}\right) + 2\cos\left(8t - \frac{\pi}{3} - \pi\right) \\ &= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t - \frac{4\pi}{3}\right) \end{aligned}$$

In the preceding expression, we could have expressed the term $2\cos\left(8t - \frac{4\pi}{3}\right)$ as $2\cos\left(8t + \frac{2\pi}{3}\right)$. Figure S6.3-3a shows amplitude and phase spectra.

(b) By inspection of the trigonometric spectra in Fig. S6.3-3a, we plot the exponential spectra as shown in Fig. S6.3-3b.

(c) By inspection of exponential spectra in Fig. S6.3-3a, we obtain

$$\begin{aligned} x(t) &= \frac{3}{2}(e^{jt} + e^{-jt}) + \frac{1}{2}\left[e^{j(5t-\frac{2\pi}{3})} + e^{-j(5t-\frac{2\pi}{3})}\right] + \left[e^{j(8t-\frac{4\pi}{3})} + e^{-j(8t-\frac{4\pi}{3})}\right] \\ &= \frac{3}{2}e^{jt} + \left(\frac{1}{2}e^{-j\frac{2\pi}{3}}\right)e^{j5t} + \left(e^{-j\frac{4\pi}{3}}\right)e^{j8t} + \frac{3}{2}e^{-jt} + \left(\frac{1}{2}e^{j\frac{2\pi}{3}}\right)e^{-j5t} + \left(e^{j\frac{4\pi}{3}}\right)e^{-j8t} \end{aligned}$$

(d) By inspection of the first line in part (c), we can immediately write $x(t)$ in the trigonometric form as

$$\begin{aligned} x(t) &= 3\cos t + \cos\left(5t - \frac{2\pi}{3}\right) + 2\cos\left(8t - \frac{4\pi}{3}\right) \\ &= 3\cos t + \sin\left(5t - \frac{\pi}{6}\right) - 2\cos\left(8t - \frac{\pi}{3}\right) \end{aligned}$$

continued ...

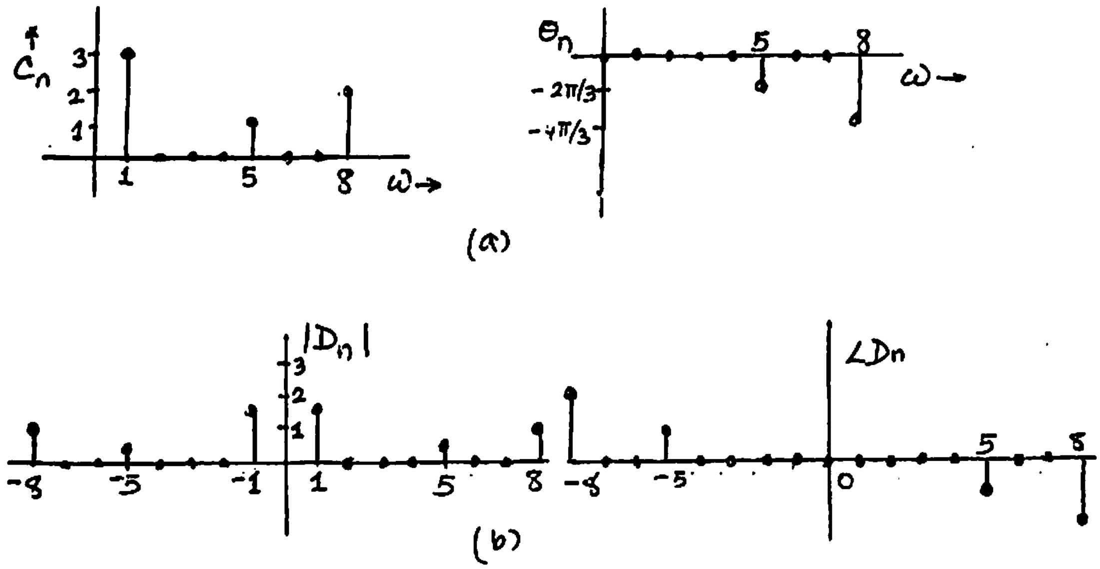


Figure S6.3-3

3. Lathi 6.4-3

$$D_n = \int_0^1 e^{-t} e^{-jn\omega_o t} dt = \frac{(e-1)(1-j2\pi n)}{e(1+4\pi^2 n^2)}$$

The transfer function of the R-C circuit is

$$H(j\omega) = \frac{1}{1+(\frac{1}{j\omega})} = \frac{j\omega}{j\omega+1}$$

The input $x(t)$ can be expressed as a Fourier series

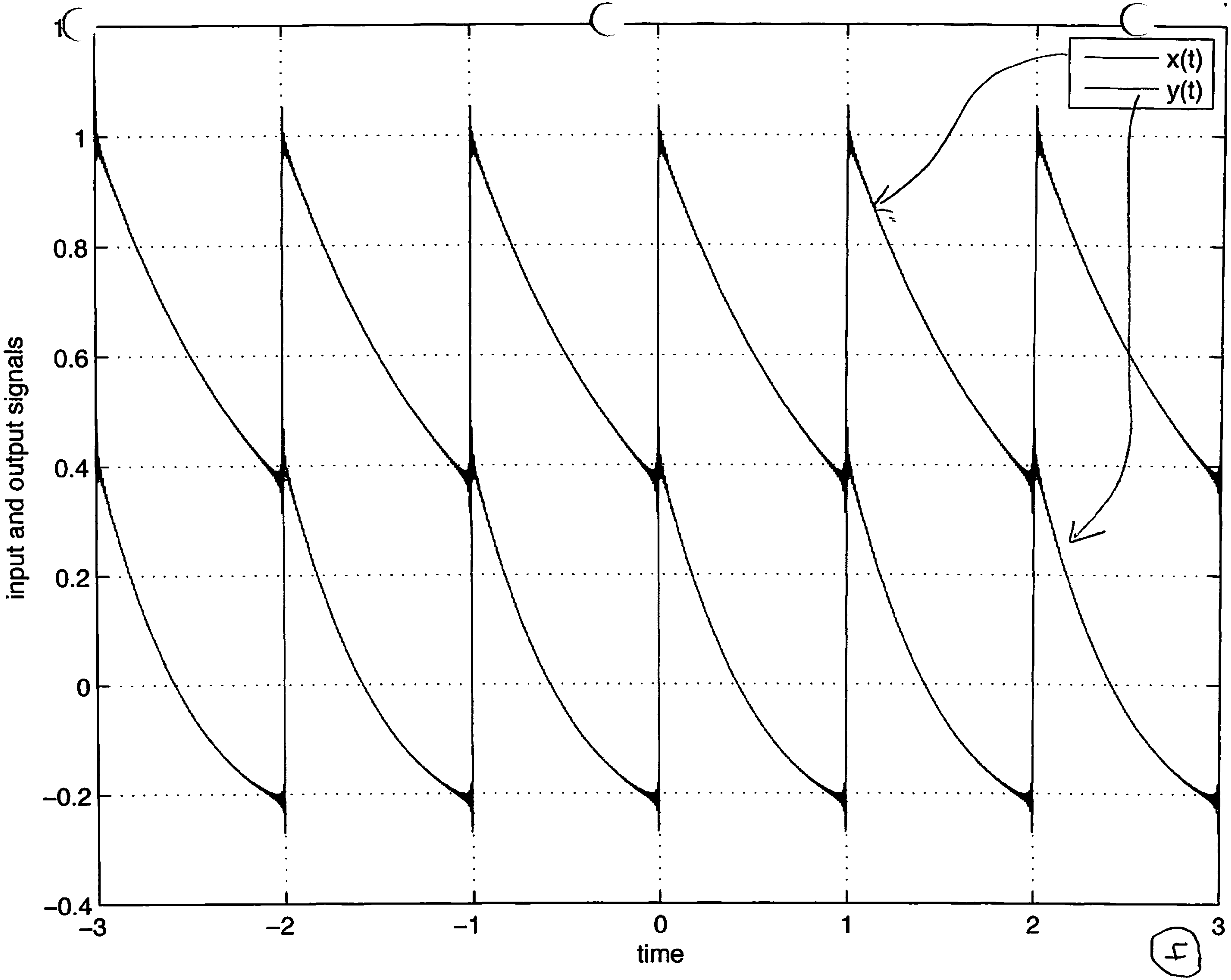
$$x(t) = \sum_{n=-\infty}^{\infty} \frac{(e-1)(1-j2\pi n)}{e(1+4\pi^2 n^2)} e^{j2\pi n t}$$

Hence the output $y(t)$ is given by

$$\begin{aligned} y(t) &= \sum_{n=-\infty}^{\infty} D_n H(j2\pi n) e^{j2\pi n t} \\ &= \sum_{n=-\infty}^{\infty} \frac{(e-1)(1-j2\pi n)(j2\pi n)}{e(1+4\pi^2 n^2)(j2\pi n+1)} e^{j2\pi n t} \end{aligned}$$

[redacted]

Matlab plot on following page...


x(t)
y(t)
input and output signals
time
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−3
−2
−1
0
1
2
3

4

```
%.ECE2311D10 Homework 4 Problem 3
% DRB 08-Apr-2010
% ==================================================
% USER PARAMETERS
% ==================================================
N = 100;
T0 = 1;
t = -3:0.001:3;
% ==================================================

j = sqrt(-1);
w0 = 2*pi/T0;
e = exp(1);

x = zeros(1,length(t));       % make array for input
y = zeros(1,length(t));       % make array for output
ybook = zeros(1,length(t));       % make array for output

for n=-N:N,

    % compute H(omega)
    omega = n*w0;
    H = j*omega/(j*omega+1);

    % compute Dn for input and Dnprime for output
    Dn = (e-1)*(1-j*w0*n)/(e*(1+w0^2*n^2));
    Dnprime = Dn*H;

    % now add to signals
    x = x+Dn*exp(j*n*w0*t);
    y = y+Dnprime*exp(j*n*w0*t);
[redacted]
end

% plot
% Note that, due to Matlab precision, there are some very small
imaginary
% components still in x and y. So we just plot the real parts here.
plot(t,real(x),t,real(y)) %,t,real(ybook));
xlabel('time');
ylabel('input and output signals');
legend('x(t)','y(t)');
grid on
```

4. Lathi 7.1-3

(a) Because $x(t) = x_o(t) + x_e(t)$ and $e^{-j\omega t} = \cos\omega t + j\sin\omega t$

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} [x_o(t) + x_e(t)] e^{-j\omega t} dt \\ &= \int_{-\infty}^{\infty} [x_o(t) + x_e(t)] \cos\omega t dt - j \int_{-\infty}^{\infty} [x_o(t) + x_e(t)] \sin\omega t dt \end{aligned}$$

Because $x_e(t)\cos\omega t$ and $x_o(t)\sin\omega t$ are even functions and $x_o(t)\cos\omega t$ and $x_e(t)\sin\omega t$ are odd functions of t, these integrals [properties in Eqs. (B.43), p. 38] reduce to

$$X(\omega) = 2\int_0^{\infty} x_e(t)\cos\omega t\, dt - 2j\int_0^{\infty} x_o(t)\sin\omega t\, dt \qquad (1)$$

Also, from the results of Prob. 7.1-1, we have

$$\mathcal{F}\{x_e(t)\} = 2\int_0^{\infty} x_e(t)\cos\omega t\, dt \quad \text{and} \quad \mathcal{F}\{x_0(t)\} = -2j\int_0^{\infty} x_o(t)\sin\omega t\, dt \quad (2)$$

From Eqs. (1) and (2), the desired result follows.

(b) We can express $u(t)$ in terms of its even and odd components as follows

$$\begin{aligned} u(t) &= \frac{1}{2}[u(t) + u(-t)] + \frac{1}{2}[u(t) - u(-t)] \\ &= \underbrace{\frac{1}{2}}_{x_e(t)} + \underbrace{\frac{1}{2}\text{sgn}(t)}_{x_o(t)} \end{aligned}$$

and

$$X_e(\omega) = \pi\delta(\omega) \qquad \text{and} \qquad X_o(\omega) = \frac{1}{j\omega}$$

Clearly, $X_e(\omega)$ is the real part and $X_o(\omega)$ is the odd part of $X(\omega)$.
We follow the same procedure for $x(t) = e^{-at}u(t)$.

$$e^{-at}u(t) = \underbrace{\frac{1}{2}[e^{-at}u(t) + e^{-at}u(-t)]}_{x_e(t)} + \underbrace{\frac{1}{2}[e^{-at}u(t) - e^{-at}u(-t)]}_{x_o(t)}$$

Also

$$X_e(\omega) = \frac{1}{2}\left[\frac{1}{j\omega + a} - \frac{1}{j\omega - a}\right] = \frac{2a}{\omega^2 + a^2}$$

and

$$X_o(\omega) = \frac{1}{2}\left[\frac{1}{j\omega + a} + \frac{1}{j\omega - a}\right] = \frac{2j\omega}{\omega^2 + a^2}$$

Clearly, $X_e(\omega)$ is the real part and $X_o(\omega)$ is the odd part of $X(\omega)$.

5. Lathi 7.1-4

(a)

$$X(\omega) = \int_0^T e^{-at} e^{-j\omega t}\, dt = \int_0^T e^{-(j\omega+a)t}\, dt = \frac{1 - e^{-(j\omega+a)T}}{j\omega + a}$$

(b)

$$X(\omega) = \int_0^T e^{at} e^{-j\omega t}\, dt = \int_0^T e^{-(j\omega-a)}\, dt = \frac{1 - e^{-(j\omega-a)T}}{j\omega - a}$$