1.

7.2-4. (a)

$$x(t) = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{-j\omega t_0}e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega(t-t_0)}\,d\omega$$
$$= \frac{1}{(2\pi)j(t-t_0)}e^{j\omega(t-t_0)}\Big|_{-\omega_0}^{\omega_0} = \frac{\sin\omega_0(t-t_0)}{\pi(t-t_0)} = \frac{\omega_0}{\pi}\text{sinc}[\omega_0(t-t_0)]$$

(b)

$$x(t) = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} je^{j\omega t}\,d\omega + \int_{0}^{\omega_0} -je^{j\omega t}\,d\omega\right]$$
$$= \frac{1}{2\pi t}e^{j\omega t}\Big|_{-\omega_0}^{0} - \frac{1}{2\pi t}e^{j\omega t}\Big|_{0}^{\omega_0} = \frac{1-\cos\omega_0 t}{\pi t}$$

2.

7.3-4. From time-shifting property

$$x(t \pm T) \Longleftrightarrow X(\omega)e^{\pm j\omega T}$$

Therefore

$$x(t+T) + x(t-T) \Longleftrightarrow X(\omega)e^{j\omega T} + X(\omega)e^{-j\omega T} = 2X(\omega)\cos\omega T$$

We can use this result to derive transforms of signals in Figure P7.3-4.

(a) Here $x(t)$ is a gate pulse as shown in Figure S7.3-4a.

$$x(t) = \text{rect}\left(\frac{t}{2}\right) \Longleftrightarrow 2\,\text{sinc}(\omega)$$

Also $T = 3$. The signal in Figure S7.3-4a is $x(t+3) + x(t-3)$, and

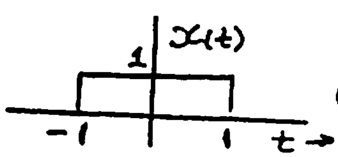
$$x(t+3) + x(t-3) \Longleftrightarrow 4\,\text{sinc}(\omega)\cos 3\omega$$

(b) Here $x(t)$ is a triangular pulse shown in Figure S7.3-4b. From the Table 4.1 (pair 19)

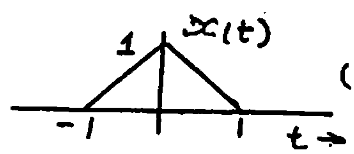
$$x(t) = \Delta\left(\frac{t}{2}\right) \Longleftrightarrow \text{sinc}^2\left(\frac{\omega}{2}\right)$$

Also $T = 3$. The signal in Figure P7.3-4b is $x(t+3) + x(t-3)$, and

$$x(t+3) + x(t-3) \Longleftrightarrow 2\,\text{sinc}^2\left(\frac{\omega}{2}\right)\cos 3\omega$$



(a)



(b)

3.

7.3-10. (a)

$$X(\omega) = \int_{-T}^{0} e^{-j\omega t}\,dt - \int_{0}^{T} e^{-j\omega t}\,dt = -\frac{2}{j\omega}[1 - \cos\omega T] = \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

(b)

$$x(t) = \text{rect}\left(\frac{t + T/2}{T}\right) - \text{rect}\left(\frac{t - T/2}{T}\right)$$

$$\begin{aligned}
\text{rect}\left(\frac{t}{T}\right) &\iff T\text{sinc}\left(\frac{\omega T}{2}\right)\\
\text{rect}\left(\frac{t \pm T/2}{T}\right) &\iff T\text{sinc}\left(\frac{\omega T}{2}\right)e^{\pm j\omega T/2}
\end{aligned}$$

$$\begin{aligned}
X(\omega) &= T\text{sinc}\left(\frac{\omega T}{2}\right)[e^{j\omega T/2} - e^{-j\omega T/2}]\\
&= 2jT\text{sinc}\left(\frac{\omega T}{2}\right)\sin\frac{\omega T}{2}\\
&= \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)
\end{aligned}$$

(c)

$$\frac{df}{dt} = \delta(t + T) - 2\delta(t) + \delta(t - T)$$

The Fourier transform of this equation yields

$$j\omega X(\omega) = e^{j\omega T} - 2 + e^{-j\omega T} = -2[1 - \cos\omega T] = -4\sin^2\left(\frac{\omega T}{2}\right)$$

Therefore

$$X(\omega) = \frac{j4}{\omega}\sin^2\left(\frac{\omega T}{2}\right)$$

4. 7.4-2. (a)

$$X(\omega) = \frac{1}{j\omega + 1} \quad \text{and} \quad H(\omega) = \frac{-1}{j\omega - 2}$$

and

$$Y(\omega) = \frac{-1}{(j\omega - 2)(j\omega + 1)} = \frac{1}{3}\left[\frac{1}{j\omega + 1} - \frac{1}{j\omega - 2}\right]$$

Therefore

$$y(t) = \frac{1}{3}\left[e^{-t}u(t) + e^{2t}u(-t)\right]$$

(b)

$$X(\omega) = \frac{-1}{j\omega - 1} \quad \text{and} \quad H(\omega) = \frac{-1}{j\omega - 2}$$

and

$$Y(\omega) = \frac{1}{(j\omega - 1)(j\omega - 2)} = \frac{-1}{j\omega - 1} - \frac{-1}{j\omega - 2}$$

Therefore

$$y(t) = [e^{t} - e^{2t}]u(-t)]$$

5.

a) $$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$= \frac{1}{RC}\left(\frac{1}{\frac{1}{RC} + j\omega}\right)$$

From line 1 of the F.T. table

$$h(t) = \frac{1}{RC}\, e^{-\frac{1}{RC}t}\, u(t)$$ (same as HW2 prob2).

b) When $x(t) = u(t)$ $\quad X(\omega) = \pi\delta(\omega) + \frac{1}{j\omega}$

(from line 11 of the table on page 702).

hence

$$Y(\omega) = H(\omega)\,X(\omega) = \frac{\pi\delta(\omega) + \frac{1}{j\omega}}{1 + j\omega RC}$$

$$= \frac{\pi\delta(\omega)}{1 + j\omega RC} + \frac{\frac{1}{j\omega}}{1 + j\omega RC}$$

Since the inverse F.T. is linear, we can compute the inverse F.T.s of these terms individually...

(continued)

$$\mathcal{F}^{-1}\left\{\frac{\pi\,\delta(\omega)}{1+j\omega RC}\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\pi\delta(\omega)}{1+j\omega RC}\,e^{j\omega t}\,d\omega \underset{\substack{\uparrow\\ \text{sifting}\\ \text{property}}}{=} \frac{\pi}{2\pi} = \frac{1}{2}$$

$$\mathcal{F}^{-1}\left\{\frac{\frac{1}{j\omega}}{1+j\omega RC}\right\} = \mathcal{F}^{-1}\left\{\left(\frac{1}{j\omega}\right)\cdot\left(\frac{1}{1+j\omega RC}\right)\right\}$$

$$= \mathcal{F}^{-1}\left\{\frac{a}{j\omega} + \frac{b}{1+j\omega RC}\right\}$$

$$a(1+j\omega RC) + b(j\omega) = 1$$

$$\Rightarrow \quad a = 1, \quad b = -RC$$

$$\mathcal{F}\left\{\frac{1}{j\omega}\right\} - \mathcal{F}\left\{\frac{RC}{1+j\omega RC}\right\} = \underbrace{\frac{1}{2}\,\mathrm{sgn}(t)}_{\substack{\uparrow\\ \text{line 12}\\ \text{of table}}} - \underbrace{e^{-\frac{1}{RC}t}\,u(t)}_{\substack{\uparrow\\ \text{line 1}\\ \text{of table.}}}$$

put it all together:

$$y(t) = \underbrace{\frac{1}{2} + \frac{1}{2}\,\mathrm{sgn}(t)}_{=u(t)} - e^{-\frac{1}{RC}t}\,u(t)$$

$$\boxed{y(t) = \left(1 - e^{-\frac{1}{RC}t}\right)u(t)}$$

same answer as HW2 problem 2.