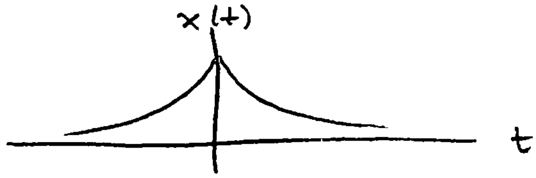
1. $x(t) = e^{-a|t|}$



Bilateral transform is necessary here because $x(t) \neq 0$ for $t<0$.

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-st} dt + \int_{0}^{\infty} e^{-at} e^{-st} dt$$

$$= \int_{-\infty}^{0} e^{-(s-a)t} dt + \int_{0}^{\infty} e^{-(s+a)t} dt$$

$$= \frac{-1}{s-a} e^{-(s-a)t} \Big|_{t=-\infty}^{t=0} + \frac{-1}{(s+a)} e^{-(s+a)t} \Big|_{t=0}^{t=\infty}$$

$$= \left[\frac{-1}{s-a} + \frac{\lim_{t\to-\infty} e^{-(s-a)t}}{s-a}\right] + \left[\frac{-\lim_{t\to\infty} e^{-(s+a)t}}{s+a} + \frac{1}{s+a}\right]$$

$$= \underbrace{\frac{1}{s+a} - \frac{1}{s-a}}_{\frac{-2a}{s^2+a^2}} + \frac{1}{s-a} \lim_{t\to-\infty} e^{-(s-a)t} - \frac{1}{s+a} \lim_{t\to\infty} e^{-(s+a)t}$$

let $s = c + j\beta$ $\quad c = Re(s) \quad \beta = Im(s)$

$$\lim_{t\to-\infty} e^{-(s-a)t} = \lim_{t\to-\infty} e^{-(c-a+j\beta)t} = \lim_{t\to-\infty} e^{-(c-a)t} e^{-j\beta t}$$

$$= \lim_{t\to\infty} e^{(c-a)t} e^{j\beta t}$$

if $c-a > 0$, $e^{(c-a)t}$ blows up $\rightarrow$ doesn't converge

if $c-a = 0$, $e^{(c-a)t} = 1$, but $e^{j\beta t}$ oscillates $\rightarrow$ doesn't converge.

if $c-a < 0$, This limit converges to zero.

The other limit can be analyzed similarly to see that it converges only if $c+a > 0$. Hence

$$\underbrace{X(s) = \frac{-2a}{s^2-a^2}}_{\text{(corrected)}} \quad \text{if } \underbrace{Re(s) > -a \text{ and } Re(s) < a}_{\text{this is the ROC.}}$$

2. Lathi 4.1-1

The unilateral Laplace transform is sufficient for both of the signals in this problem because both signals are zero for all $t<0$.

(a)

$$x(t) = u(t) - u(t-1)$$

$$\begin{aligned} X(s) = \int_0^1 e^{-st}\,dt &= -\frac{e^{-st}}{s}\Big|_0^1 \\ &= -\frac{1}{s}[e^{-s}-1] \\ &= \frac{1}{s}[1-e^{-s}] \end{aligned}$$

Note that the result is valid for all values of s; hence the region of convergence is the entire s-plane. The abscissa of convergence is $\sigma_0 = -\infty$.

(d)

$$x(t) = (e^{2t} - 2e^{-t})u(t)$$

$$\begin{aligned} X(s) &= \int_0^\infty (e^{2t} - 2e^{-t})e^{-st}\,dt \\ &= \int_0^\infty e^{2t}e^{-st}\,dt - 2\int_0^\infty e^{-t}e^{-st}\,dt \\ &= \int_0^\infty e^{-(s-2)t}\,dt - 2\int_0^\infty e^{-(s+1)t}\,dt \\ &= \frac{1}{s-2} - \frac{2}{s+1} \end{aligned}$$

We get the first term only if Re $s > 2$, and we get the second term only if Re $(s) > -1$. Both conditions will be satisfied if Re $(s) > 2$ or $\sigma_0 > 2$. Hence:

$$X(s) = \frac{1}{s-2} - \frac{2}{s+1} \qquad \text{for} \quad \sigma_0 > 2$$

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3. Lathi 4.1-3

(a)

$$\begin{aligned} X(s) &= \frac{2s+5}{s^2+5s+6} = \frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3} \\ x(t) &= (e^{-2t}+e^{-3t})u(t) \end{aligned}$$

(c)

$$X(s) = \frac{(s+1)^2}{s^2-s-6} = \frac{(s+1)^2}{(s+2)(s-3)}$$

This is an improper fraction with $b_n = b_2 = 1$. Therefore

$$\begin{aligned} X(s) &= 1 + \frac{a}{s+2} + \frac{b}{s-3} = 1 - \frac{0.2}{s+2} + \frac{3.2}{s-3} \\ x(t) &= \delta(t) + (3.2e^{3t} - 0.2e^{-2t})u(t) \end{aligned}$$

4. Lathi 4.2-9

(a) Using the differentiation in s property,

$$\mathcal{L}[tx(t)] = -\frac{d}{ds}X(s).$$

(b) $y(t) = tx(t) = t\frac{1}{t}u(t) = u(t)$. Thus, $Y(s) = \int_{-\infty}^{\infty} u(t)e^{-st}dt = \int_0^{\infty} e^{-st}dt = \left.\frac{e^{-st}}{-s}\right|_{t=0}^{\infty}$. For $\sigma > 0$, this simplifies to $Y(s) = \frac{1}{s}$.

(c) Combining the previous two parts yields $-\frac{d}{ds}X(s) = \frac{1}{s}$. Thus,

$$X(s) = -\int \frac{1}{s}ds = -\ln(s).$$

5. Lathi 4.4-1

Figure S4.4-1 shows the transformed network. The loop equations are

$$\begin{aligned}
(1+\frac{1}{s})Y_1(s) - \frac{1}{s}Y_2(s) &= \frac{1}{(s+1)^2} \\
-\frac{1}{s}Y_1(s) + (s+1+\frac{1}{s})Y_2(s) &= 0
\end{aligned}$$

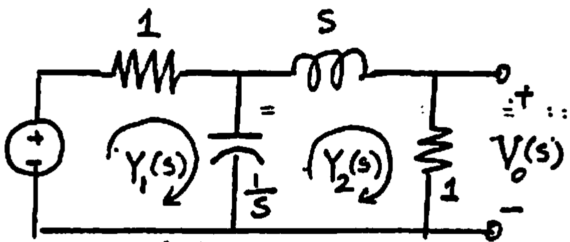
$$\begin{bmatrix} \frac{s+1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & \frac{s^2+s+1}{s} \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{(s+1)^2} \\ 0 \end{bmatrix}$$

Cramer's rule yields

$$Y_2(s) = \frac{1}{(s+1)^2(s^2+2s+2)} = \frac{1}{(s+1)^2} - \frac{1}{s^2+2s+2}$$

see next page for diff eq $\longrightarrow$

$$v_0(t) = y_2(t) = (te^{-t} - \frac{1}{2}e^{-t}\sin t)u(t)$$



S4.4-1

6. Lathi 4.4-3

The impedance seen by the source $x(t)$ is

$$Z(s) = \frac{Ls(1/Cs)}{Ls+(1/Cs)} = \frac{Ls}{LCs^2+1} = \frac{Ls\omega_0^2}{s^2+\omega_0^2}$$

The current $Y(s)$ is given by

$$Y(s) = \frac{X(s)}{Z(s)} = \underbrace{\frac{s^2+\omega_0^2}{Ls\omega_0^2}}_{H(s)} X(s)$$

(a)

$$X(s) = \frac{As}{s^2+\omega_0^2}, \qquad Y(s) = \frac{A}{L\omega_0^2} \quad \text{and} \quad y(t) = \frac{A}{L\omega_0^2}\delta(t)$$

(b)

$$X(s) = \frac{A\omega_0}{s^2+\omega_0^2}, \qquad Y(s) = \frac{A}{L\omega_0 s} \quad \text{and} \quad y(t) = \frac{A}{L\omega_0}u(t)$$

This system is not BIBO stable because, in part a, a bounded input resulted in an unbounded output. Also, the only pole of H(s) occurs at s=0, which says that the system can't be BIBO stable. This system is <u>marginally</u> stable, but not asymptotically stable.

[Corrected 3-May-2010]

problem 5 continued...

$Y(s) = H(s)\,X(s)$

In this problem, $H(s) = \dfrac{1}{s^2+2s+2}$ and $X(s) = \dfrac{1}{(s+1)^2}$

Hence

$$(s^2+2s+2)\,Y(s) = X(s)$$

use time differentiation property with relaxed initial conditions to write

$$\boxed{\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + 2y(t) = x(t)}$$

This is the differential equation that describes the dynamics of the system.