ECE4304 Homework Assignment Number 1

Due by 4:50pm on Thursday 18-Jan-2007

1 Required Reading

- Haykin Appendix 1 and Chapter 1.1-1.4

2 Problems

80 points total. You must show all of your work and your work must be neat to receive credit for a problem. Complete the following problems:

1. 12 points. A random variable X has a probability density function

$$f_X(x) = c\exp(-\alpha x) \qquad 0 \le x < \infty.$$

   (a) Find the appropriate value for c (it will be a function of α).

   (b) Compute $F_X(x)$ as a function of α and x and plot $F_X(x)$ versus x for $\alpha = 1$.

   (c) Compute $\mathrm{E}[X]$.

   (d) Compute $\mathrm{var}[X]$.

2. 10 points. Suppose you have a wired communication system where the transmitter puts +1Vdc on the communication circuit if a binary one is transmitted and puts -1Vdc on the circuit if a binary zero is transmitted. Binary ones and zeros are sent with equal probability. The receiver decides if the transmission was a one or a zero by sampling the voltage in the circuit and looking at the sign of this sample.

   Unfortunately, there is a lot of crosstalk with nearby communication circuits and the signal at the receiver is corrupted by noise. The sampled voltage at the receiver can be modeled as a random variable $X = V + W$ where V is the voltage applied to the circuit by the transmitter and W is a Guassian random variable with zero mean and a standard deviation of 0.3 volts. *What is the probability that the receiver makes an incorrect decision in this scenario?* Hint: You should consider the case when a zero was sent but a one was decided. You should also consider the case when a one was sent but a zero was decided. Using principles of conditional probability (See Haykin Appendix 1.1), put these results together to form your final answer. Another hint: you might find Matlab functions "erf" and/or "erfc" handy.

3. 18 points. Two random variables X and Y have the joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} c(x-y)^2 & -1 \le x \le 1 \text{ and } -1 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the appropriate value for c.

(b) Compute $F_{X,Y}(x, y)$.

(c) Compute the marginal pdfs $f_X(x)$ and $f_Y(y)$.

(d) Compute $\mathrm{P}[X > 0, Y > 0]$.

(e) Compute $\mathrm{corr}[X, Y]$.

(f) Compute $\mathrm{cov}[X, Y]$.

4. 10 points. Haykin Problem 1.4.

5. 15 points. Haykin Problem 1.6.

6. 15 points. In this problem, you will write a Matlab simulation to verify your answer to Problem 2. Use the Matlab function `randn` to generate a vector containing at least 1E5 independent Gaussian random variables, each with zero mean and standard deviation equal to 0.3. Use the command `doc randn` to get extensive help information on how to use the `randn` function. Generate another vector of the same length containing equally probable values of -1 and +1. This vector will represent the voltage applied to the communication circuit by the transmitter. Add your signal and noise vectors together to produce a vector of noisy samples at the receiver. Use the `sign` function in Matlab to generate decisions at the receiver and then count up the total number of incorrect decisions. Hint: An easy way to count up the number of different elements between vectors X and Y in Matlab is `sum(X =Y)`. Use this result to estimate the probability of incorrect decision. Does your simulation agree with your analytical results? Play around with this to see the effect of transmitter voltage on the error probability. What if the transmitter sent +2Vdc for a binary one and -2Vdc for a binary zero?

   Please hand in your commented Matlab code and discuss your results. If you generate any plots, label all plot axes.