ECE4304 Homework Assignment Number 2

Due by 4:50pm on Wednesday 24-Jan-2007

1 Required Reading

- Haykin Chapter 1.5-1.13 and Chapter 2.10-2.12

2 Problems

80 points total. You must show all of your work and your work must be neat to receive credit for a problem. Complete the following problems:

1. 15 points. Haykin 1.14. Additionally, compute the total power of the random process at the output of the filter.

2. 10 points. Suppose $X(t)$ is a WSS white Gaussian random process with $S_X(f) = 10^{-6}$ watts/Hz for all $\infty < f < \infty$. This random process is filtered by an ideal low-pass filter with frequency response

$$H(f) = \begin{cases} 1 & -W < f < W \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Compute the PSD of random process at the output of the low-pass filter.
   (b) Compute the autocorrelation function of random process at the output of the low-pass filter.
   (c) Compute the total power of the random process at the output of the low-pass filter using the PSD method and the autocorrelation function method. Do both methods give the same result?

3. 15 points. Haykin 1.26 parts a–b (skip part c).

4. 10 points. The power spectral density of a narrowband random process $X(t)$ is given in Figure 1. Compute the total power in this random process. Find the power spectral densities of the in-phase and quadrature components of $X(t)$. Find their cross spectral densities.

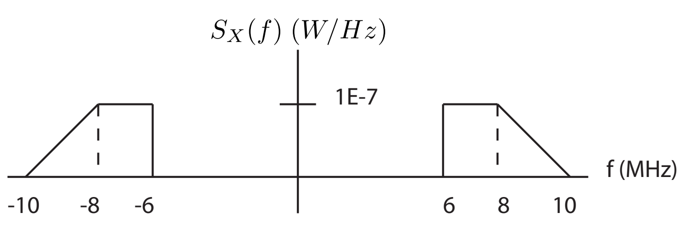

Figure 1: Power spectral density of the random process $X(t)$.

5. 30 points total. In this problem, you will compare the performance of two AM communication systems. In both systems, the message signal $M(t)$ is assumed to be a WSS random process limited to $-1 < M(t) < 1$ for all t with power spectral density shown in Figure 2.

   (a) 5 points. Compute the amount of power in the message signal $M(t)$.

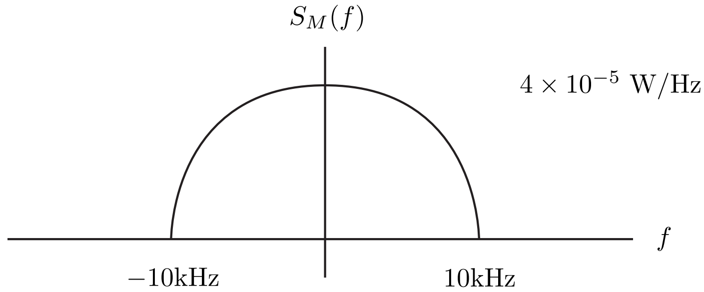

Figure 2: Power spectral density of message signal. Note that $S_M(f) = 4 \times 10^{-5} \times \left(1 - \left(\frac{f}{10^4}\right)^2\right)$ W/Hz.

(b) 10 points. Suppose you have a DSB-SC AM communication system where the modulated transmission is given as

$$u(t) \quad = \quad A_c M(t) \cos(2\pi f_c t + \phi_c)$$

where A_c is the carrier amplitude, f_c is the carrier frequency, and ϕ_c is the uniformly distributed carrier phase on the interval $[-\pi, \pi)$ independent of the message signal. Determine the amount of transmit power required to achieve 30dB output SNR assuming that

- the carrier frequency: f_c =580kHz.
- the channel attenuation is -70dB,
- the noise at the input of the receiver is white with PSD $S_N(f) = \frac{N_0}{2} = 10^{-12}$ W/Hz, and
- the receiver is the standard DSB-SC receiver with bandwidth $B = 10$kHz.
- the receiver can perfectly recover the phase of the carrier, i.e. $\phi = \phi_c$.

(c) 10 points. Suppose you have a *conventionally modulated* DSB AM communication system where the modulated transmission is given as

$$u(t) \quad = \quad A_c[1 + 0.9M(t)] \cos(2\pi f_c t + \phi_c)$$

where all quantities are defined in part (b). Determine the amount of transmit power required to achieve 30dB output SNR assuming that

- the carrier frequency: f_c =580kHz.
- the channel attenuation is -70dB,
- the noise at the input of the receiver is white with PSD $S_N(f) = \frac{N_0}{2} = 10^{-12}$ W/Hz, and
- the receiver is the standard conventionally modulated DSB AM receiver with bandwidth $B = 10$kHz (see Figure 3).
- the receiver can perfectly recover the phase of the carrier, i.e. $\phi = \phi_c$.

(d) 5 points. Compare your results from parts (a) and (b) and comment.

For reference, Figure 3 is a conceptual block diagram of the conventional AM receiver. Note that the DSB-SC AM receiver is identical except that it does not have the DC blocking device.

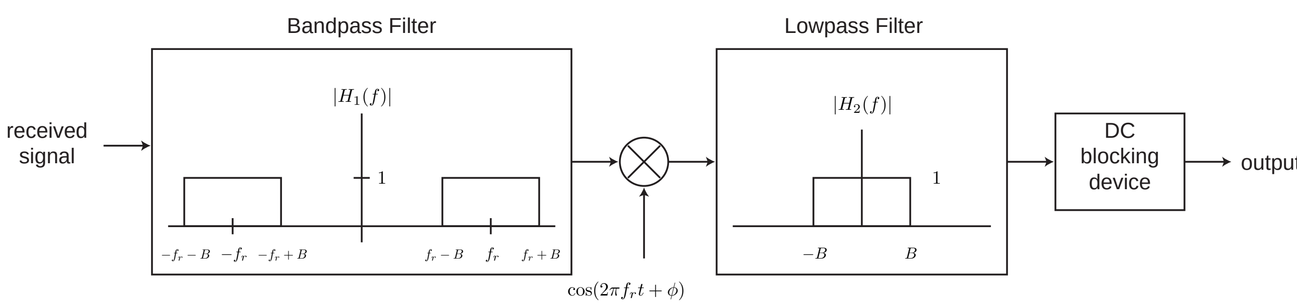

Figure 3: Conventional AM receiver.