1. \[ R_X(z) = \exp \left(-2\nu|z|\right) \]

\[ S_X(f) = \int_{-\infty}^{\infty} R_X(z) \exp(-j2\pi ft) dz \]

\[ = \int_{-\infty}^{0} \exp(2\nu z) \exp(-j2\pi ft) dz + \int_{0}^{\infty} \exp(2\nu z) \exp(-j2\pi ft) dz \]

\[ = \frac{1}{2(\nu - j\pi f)} \left[ \exp(2\nu z - j2\pi ft) \right] \bigg|_0^\infty - \frac{\exp(-2\nu z - j2\pi ft)}{2(\nu + j\pi f)} \bigg|_0^\infty \]

\[ = \frac{\nu}{\nu^2 + \pi^2 f^2} \]

From the plot, we know the transfer function of the filter is

\[ H(f) = \frac{1}{1 + j2\pi f RC} \]

Hence

\[ S_Y(f) = \left| H(f) \right|^2 S_X(f) = \frac{\nu}{1 + (2\pi f RC)^2 + \pi^2 f^2} \]

\[ = \frac{\nu}{1 - 4\nu^2 c^2 \nu^2} \left[ - \frac{1}{(1/2RC)^2 + \pi^2 f^2} + \frac{1}{\nu^2 + \pi^2 f^2} \right] \]

\[ \frac{1/2RC}{(1/2RC)^2 + \pi^2 f^2} \leftrightarrow \exp(-1t/RC) \]

\[ \frac{\nu}{\nu^2 + \pi^2 f^2} \leftrightarrow \exp\left(-2\nu|t|\right) \]

So

\[ R_Y(z) = \frac{\nu}{1 - 4\nu^2 c^2 \nu^2} \left[ \frac{1}{\nu} \exp\left(-2\nu(z) - 2RC\exp(-|z|/RC)\right) \right] \]
\[ \text{Power} = R_Y(10) = \frac{V^2}{1 - 4RC^2V^2} \left[ \frac{1}{V^2} - 2RC \right] = \frac{1}{1 + 2RCV} \]

2. a) \[ S_Y(f) = |H(f)|^2 S_X(f) = \begin{cases} 10^{-6}, & -\omega < f < \omega \\ 0, & \text{otherwise} \end{cases} \]

b) \[ R_Y(z) = 2 \times 10^{-6} \omega \text{sinc}(2\omega z) \]

c) \[ P_{\text{psd}} = \int_{-\omega}^{\omega} S_Y(f) \, df = 2 \times 10^{-6} \omega \]

\[ P_R = R_Y(10) = 2 \times 10^{-6} \omega \text{sinc}(10) = 2 \times 10^{-6} \omega \]

\[ P_{\text{psd}} = P_R \]

3. a)

Let \( S_{\text{nbp}}(f) \) denote the PSD of \( n_{\text{bp}}(t) \), we have

\[ N_0^2 \]

\[ S_{\text{nbp}}(f) \]

\[ 0 \]

\[ f_1 \]

\[ 2f_0 \]
let \( S_{Nm}(f) \) denote the PSD of \( N_m(t) \), we have

\[
S_{Nm}(f) = \frac{1}{2} \left( S_{NbP}(f-f_c) + S_{NbP}(f+f_c) \right)
\]

After the LPF, only the part between \(-B\) and \(B\) left.

So \( S_N(f) = \begin{cases} 
\frac{N_0}{4} & -B \leq f \leq B \\
0 & \text{otherwise}
\end{cases} \)

\[
R_n(z) = 2 - \frac{N_0}{4} B \text{sinc}(2Bz) = \frac{N_0}{2} B \text{sinc}(2Bz)
\]

b) \( E[n(t)] = 0 \)

\[
\text{Var}[n(t)] = E[n(t)^2] = \int_{-B}^{B} \frac{N_0}{4} \, df = \frac{N_0 B}{2}
\]

Problem 4 is on the last page...

5. a) Power = \( \int_{-10k}^{10k} S_n(f) \, df = \int_{-10k}^{10k} 4\times10^{-5} \times (1 - \left( \frac{f}{10k} \right)^2) \, df \)

\[= \frac{8}{15} \text{W} \]

b) using my notes in class:

\[
\text{SNR} = \frac{\chi^2 A_c^2 P_m \cos^2(\phi_c - \phi)}{2BN_0} = 10^3
\]

\[
\chi^2 = 10^{-7}
\]

\[
N_0 = 2\times10^{-12} \text{ W/Hz}
\]

\[
B = 10k \text{ Hz}
\]

\[
\phi = \phi_c
\]

\[
\Rightarrow A_c^2 P_m = 4\times10^2
\]

transmit power is

\[
P_t = E[(M(t)A_c\cos(2\pi f_c t + \phi_c))^2]
\]

\[= \frac{1}{2} A_c^2 P_m E[1 + \cos(4\pi f_c t + 2\phi_c)]
\]

\[= \frac{1}{2} A_c^2 P_m = 2\times10^2 \text{ W}
\]
\[ (C) \quad u(t) = A_c \left[ 1 + 0.9M(t) \right] \cos(2\pi f_c t + \phi_c) \]
\[ y(t) = x A_c \left[ 1 + 0.9M(t) \right] \cos(2\pi f_c t + \phi_c) + N_{bp}(t) \]
\[ y(t) = \frac{1}{2} x A_c \left[ 0.9M(t) \right] \cos(\phi_c - \phi) + LPF[\cos(2\pi f_c t + \phi)N_{bp}(t)] \]

Power of the signal part = \[ E \left[ \left( \frac{1}{2} x A_c \quad 0.9M(t) \quad \cos(\phi_c - \phi) \right)^2 \right] \]
\[ = \frac{1}{4} x^2 A_c^2 \cos^2(\phi_c - \phi) \times 0.81 \text{Pm} \]

Power of the noise part is the same as the notes.

So \[ \text{SNR} = \frac{x^2 A_c^2 \cos^2(\phi_c - \phi)}{2B_{10}} \]

\[ \Rightarrow \quad \frac{10^{-7} A_c^2 \left( 0.81 \text{Pm} \right)}{2 \times 10 \times 8 \times 10^{-12}} = 10^3 \]
\[ \text{Pm} = \frac{8}{15} \text{w} \]
\[ \Rightarrow \quad A_c^2 = 925.93 \]

\[ P_t = E \left[ x A_c \left[ 1 + 0.9M(t) \right] \cos(2\pi f_c t + \phi_c) \right]^2 \]
\[ = \frac{1}{2} A_c^2 E \left[ 1 + 0.9M(t) \right]^2 (1 + E[\cos(4\pi f_c t + 2\phi_c)]) \]
\[ = \frac{1}{2} A_c^2 (1 + 0.81 \text{Pm}) \]
\[ = 663 \text{ w} \]

(d) From the results of (b) and (c), we notice that conventional DSB AM needs more transmission power. Because it needs power to transmit carrier. However, carrier does not carry any message. So we can use DSB-SC to transmit the same info with less power.
4. \( P_{\text{total}} = \int_{-\infty}^{\infty} S_x(f) \, df \)

\[ = 2 \times \left( \frac{2 \times 10^6 \times 10^{-7}}{2} + 2 \times 10^6 \times 10^{-7} \right) \]

\[ = 0.6 \text{ W} \]

\( S_{x_1}(f) = S_{x_0}(f) = \begin{cases} S_x(f-f_c) + S_x(f+f_c), & -2 \leq f \leq 2 \text{ otherwise} \\ 0 & \text{otherwise} \end{cases} \)

\( S_{x_2}(f) = S_{x_0}(f) = \begin{cases} 2 \times 10^{-7} \left( 1 - \frac{|f|}{4} \right), & -2 \leq f \leq 2 \\ 0 & \text{otherwise} \end{cases} \)

\( S_{1x}(f) = \begin{cases} \int [S_N(f+f_c) - S_N(f-f_c)] & -2 \leq f \leq 2 \\ 0 & \text{otherwise} \end{cases} \)

\( \Rightarrow S_{2x}(f) = \begin{cases} -j \times 0.5 \times 10^{-7} f & -2 \leq f \leq 2 \\ 0 & \text{otherwise} \end{cases} \)