ECE4703 Exam Number 3

Your Name: ______________________________ Your box #: __________

December 12, 2006

Tips:

- Look over all of the questions before starting.
- Budget your time to allow yourself some time to work on each question.
- Write neatly!
- This exam is worth a total of 100 points.
- Attach your "cheat sheet" to the exam when you hand it in.

problem 1	problem 2	problem 3	total exam 3 score
20 points	30 points	50 points	100 points

1. 20 points. Suppose you are designing an algorithm that requires computation of a 8192-point DFT/FFT. You first write code to compute the 8192-point DFT and determine that it takes 100ms for this code to run. Estimate the time it would take to compute a 8192-point FFT on this same platform.

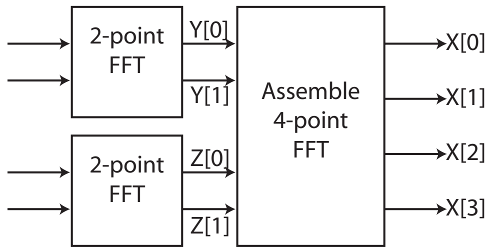
2. 30 points total. Suppose you wish to compute the 4-point FFT of the array

$$\boldsymbol{x} = \{x[0], x[1], x[2], x[3]\}.$$

The output of the FFT is denoted as

$$\boldsymbol{X} = \text{FFT}(\boldsymbol{x}) = \{X[0], X[1], X[2], X[3]\}.$$

As shown in the figure below, you know that you need to call a 2-point FFT function twice in order to compute the 4-point FFT. Denote the output of the first 2-point FFT call as $\boldsymbol{Y} = \{Y[0], Y[1]\}$ and the output of the second 2-point FFT call as $\boldsymbol{Z} = \{Z[0], Z[1]\}$.



Given

$$\begin{aligned} x[0] &= 0 \\ x[1] &= 1 \\ x[2] &= 2 \\ x[3] &= 3, \end{aligned}$$

compute $Y[0], Y[1], Z[0], Z[1], X[0], X[1], X[2]$, and $X[3]$. Show your work and explain your reasoning.

3. 50 points total. Suppose you have an LMS-adapted FIR filter with 1 coefficient ($K = 1$) denoted as $b[0, n]$, where n denotes the time index, and you are using this filter in a system identification application. Suppose the input signal to the unknown system $x[n]$ is a white noise signal with mean squared value of $\mathrm{E}[x^2[n]] = 0.4$.

(a) 10 points. What is the maximum value of the LMS step-size μ that will ensure that the LMS filter will converge in this case?

(b) 20 points. Suppose you observe the following sequence of inputs and outputs from the unknown system.

time index n	0	1	2	3	4	5	6	7	8	9	...
input $x[n]$	1	-0.5	-2	-0.8	3	0.3	-1	1.5	1.1	-1.8	...
unknown system output $d[n]$	-4	2	8	3.2	-12	-1.2	4	-6	-4.4	7.2	...

Given an initial filter coefficient value of $b[0, 0] = 0$ and an LMS step size $\mu = 0.5$, fill in the following table. Note that $y[n]$ denotes the adaptive filter output at time n and $e[n]$ denotes the difference between the unknown system output $d[n]$ and the adaptive filter output $y[n]$ at time n.

time index n	0	1	2
$b[0, n]$	0		
$y[n]$			
$e[n]$			

(c) 20 points. Assuming the trend for $x[n]$ and $d[n]$ continues, can you predict the value of $b[0, n]$ as $n \to \infty$? Hint: Given what you know about $x[n]$ and $d[n]$, what choice for $b[0, n]$ minimizes $\mathrm{E}[e^2[n]]$? What can you say about $e^2[n]$ as $n \to \infty$? Explain.