ECE4703: Lecture 8

D. Richard Brown III

Worcester Polytechnic Institute

30-Nov-2009

FFT Complexity Analysis: $N = 1$

DFT equation:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \qquad \text{for } k = 0, 1, \dots, N-1$$

When $N = 1$, the FFT and the DFT do the same computations and we simply have

$$X[0] = x[0]e^0$$

No multiplies or additions. Hence a one-point DFT/FFT has no MACs.

FFT Complexity Analysis: $N = 2$

Two-point FFT computed using radix-2 decimation in time:

$$\begin{aligned} X[0] &= X_{even}[0] + e^{-j2\pi\cdot 0/2} X_{odd}[0] = X_{even}[0] + X_{odd}[0] \\ X[1] &= X_{even}[1] + e^{-j2\pi\cdot 1/2} X_{odd}[1] = X_{even}[1] - X_{odd}[1] \end{aligned}$$

Note that $X_{even}[k]$ and $X_{odd}[k]$ for $k = 0, 1$ are just one-point FFTs.

$$\begin{aligned} X_{even}[0] &= \text{DFT}\{x[0]\} = x[0] \\ X_{odd}[0] &= \text{DFT}\{x[1]\} = x[1] \\ X_{even}[1] &= \text{DFT}\{x[0]\} = x[0] \\ X_{odd}[1] &= \text{DFT}\{x[1]\} = x[1] \end{aligned}$$

Hence,

$$\begin{aligned} X[0] &= x[0] + x[1] \\ X[1] &= x[0] - x[1] \end{aligned}$$

How many MACs in the R2-DIT two-point FFT?

FFT Complexity Analysis: $N = 4$ (part 1)

Four-point FFT computed using radix-2 decimation in time:

$$
\begin{aligned}
X[0] &= X_{even}[0] + e^{-j2\pi\cdot 0/4}X_{odd}[0] = X_{even}[0] + X_{odd}[0] \\
X[1] &= X_{even}[1] + e^{-j2\pi\cdot 1/4}X_{odd}[1] = X_{even}[1] - jX_{odd}[1] \\
X[2] &= X_{even}[2] + e^{-j2\pi\cdot 2/4}X_{odd}[2] = X_{even}[2] - X_{odd}[2] \\
X[3] &= X_{even}[3] + e^{-j2\pi\cdot 3/4}X_{odd}[3] = X_{even}[3] + jX_{odd}[3]
\end{aligned}
$$

Remarks:

- $X_{even}[k]$ and $X_{odd}[k]$ for $k = 0, 1, 3, 4$ are all two-point FFTs.
- Recall that an N-point DFT/FFT is periodic in the sense that $X[k] = X[k + N]$.
- These facts imply that $X_{even}[2] = X_{even}[0]$, $X_{even}[3] = X_{even}[1]$, $X_{odd}[2] = X_{odd}[0]$, and $X_{odd}[3] = X_{odd}[1]$.
- Hence, we only need to compute four two-point FFTs: $X_{even}[0]$, $X_{even}[1]$, $X_{odd}[0]$, and $X_{odd}[1]$.

FFT Complexity Analysis: $N = 4$ (part 2)

Four-point FFT computed using radix-2 decimation in time:

$$
\begin{aligned}
X[0] &= X_{even}[0] + e^{-j2\pi\cdot 0/4}X_{odd}[0] = X_{even}[0] + X_{odd}[0] \\
X[1] &= X_{even}[1] + e^{-j2\pi\cdot 1/4}X_{odd}[1] = X_{even}[1] - jX_{odd}[1] \\
X[2] &= X_{even}[2] + e^{-j2\pi\cdot 2/4}X_{odd}[2] = X_{even}[2] - X_{odd}[2] \\
X[3] &= X_{even}[3] + e^{-j2\pi\cdot 3/4}X_{odd}[3] = X_{even}[3] + jX_{odd}[3]
\end{aligned}
$$

Computation of $X_{even}[0]$ and $X_{even}[1]$ requires how many MACs? 2

Computation of $X_{odd}[0]$ and $X_{odd}[1]$ requires how many MACs? 2

How many more MACs are required to assemble the two two-point FFTs into a four-point FFT? 4

Hence, the total MACs needed to compute a four-point FFT is 8.

FFT Complexity Analysis: $N = 8$

Eight-point FFT computed using radix-2 decimation in time follows the same accounting:

$$X[k] = X_{even}[k] + e^{-j2\pi \cdot k/8} X_{odd}[k] \qquad \text{for } k = 0, 1, \dots, 8$$

Computation of $X_{even}[k]$ for $k = 0, 1, 2, 3$ requires how many MACs? 8

Computation of $X_{odd}[k]$ for $k = 0, 1, 2, 3$ requires how many MACs? 8

How many more MACs are required to assemble the two four-point FFTs into an eight-point FFT? 8

Hence, the total MACs needed to compute an eight-point FFT is 24.

FFT Complexity Analysis: General N

Operation	MACs
Computation of N one-point FFTs	0
Assembling $\frac{N}{2}$ two-point FFTs from N one-point FFTs	N
Assembling $\frac{N}{4}$ four-point FFTs from $\frac{N}{2}$ two-point FFTs	N
Assembling $\frac{N}{8}$ eight-point FFTs from $\frac{N}{4}$ four-point FFTs	N
$\vdots$	$\vdots$
Assembling one N-point FFT from two $\frac{N}{2}$-point FFTs	N

Hence, the total MACs needed to compute an N-point FFT is ______.