ECE503 Midterm Bonus Exam

Your Name: SOLUTION Your box #: ____________

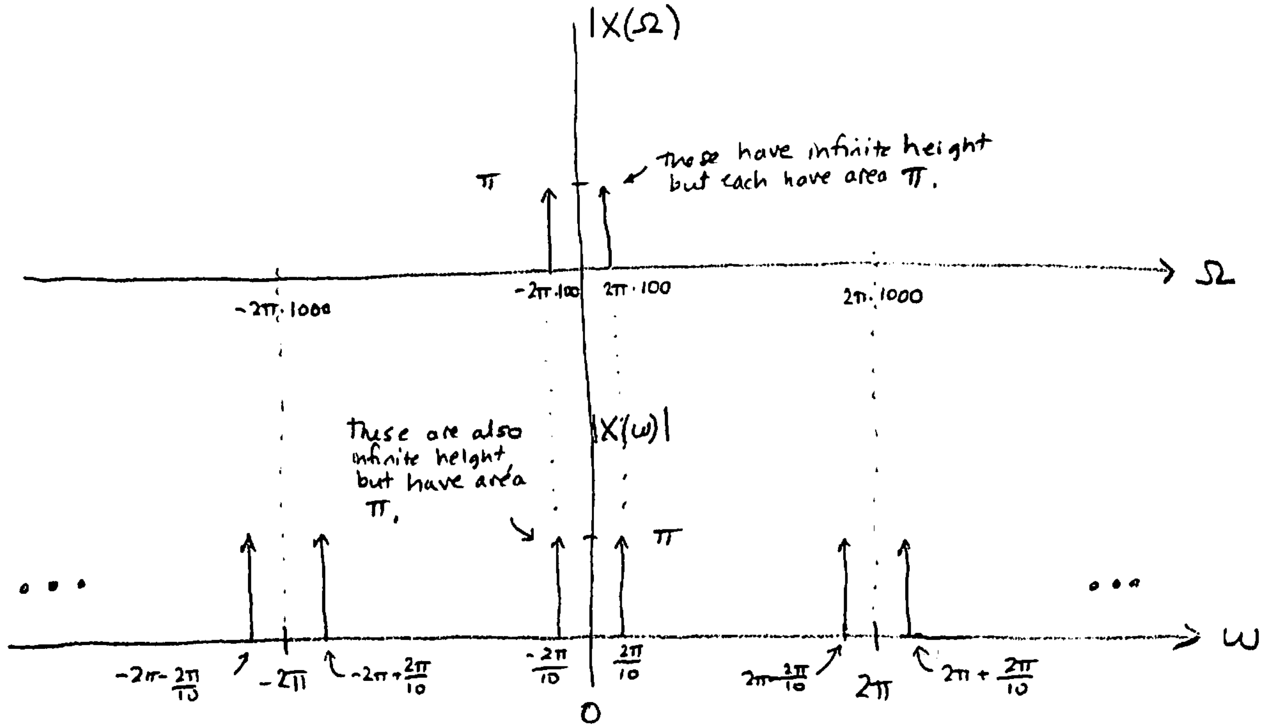
April 2, 2012

Tips:

- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work! Points will be deducted for a disorderly presentation of your solution.
- This exam is worth a total of 60 points.
- This exam is to be completed in 60 minutes.
- You are permitted to consult your textbook, one handwritten "cheat sheet", and a calculator.
- Attach your "cheat sheet" to the exam when you hand it in.

problem 1	problem 2	problem 3	total midterm bonus exam score
20 points	20 points	20 points	60 points

1. 20 points. A continuous-time signal $x(t) = \cos(2\pi 100 t)$ for all $-\infty < t < \infty$ is ideally sampled at $F_T = 1000$ Hz to get a discrete-time sequence $\{x[n]\}$ for $n = \ldots, -1, 0, 1, \ldots$. Neatly sketch the magnitude of $X(\Omega) = \mathrm{CTFT}(x(t))$ and the magnitude of $X(\omega) = \mathrm{DTFT}(x[n])$.



$$x(t) = \tfrac{1}{2} e^{j\Omega_0 t} + \tfrac{1}{2} e^{-j\Omega_0 t} \xleftrightarrow{\text{CTFT}} X(\Omega) = \pi\delta(\Omega - \Omega_0) + \pi\delta(\Omega_0 + \Omega_0)$$

$$x[n] = \cos(\omega_0 n) = \tfrac{1}{2} e^{j\omega_0 n} + \tfrac{1}{2} e^{-j\omega_0 n} \xleftrightarrow{\text{DTFT}} \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + k2\pi) + \pi \sum_{k=-\infty}^{\infty} \delta(\omega + \omega_0 + k2\pi)$$

<full credit will be given on this problem if the sketch is neat and accurate — the areas under the delta-functions are not critical>

Note $X(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \bar{X}\left(\frac{\omega + k2\pi}{T}\right)$

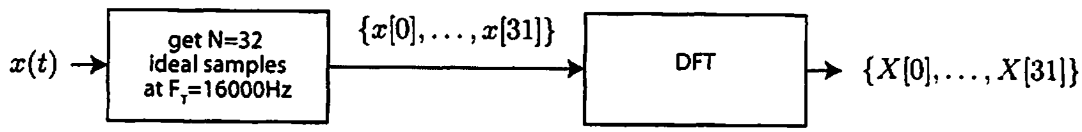
$$\bar{X}\left(\frac{\omega + k2\pi}{T}\right) = \pi\delta\left(\frac{\omega + k2\pi}{T} - \Omega_0\right) + \pi\delta\left(\frac{\omega + k2\pi}{T} + \Omega_0\right)$$

$$= \pi\delta\left(\frac{\omega - \omega_0 + k2\pi}{T}\right) + \pi\delta\left(\frac{\omega + \omega_0 + k2\pi}{T}\right)$$

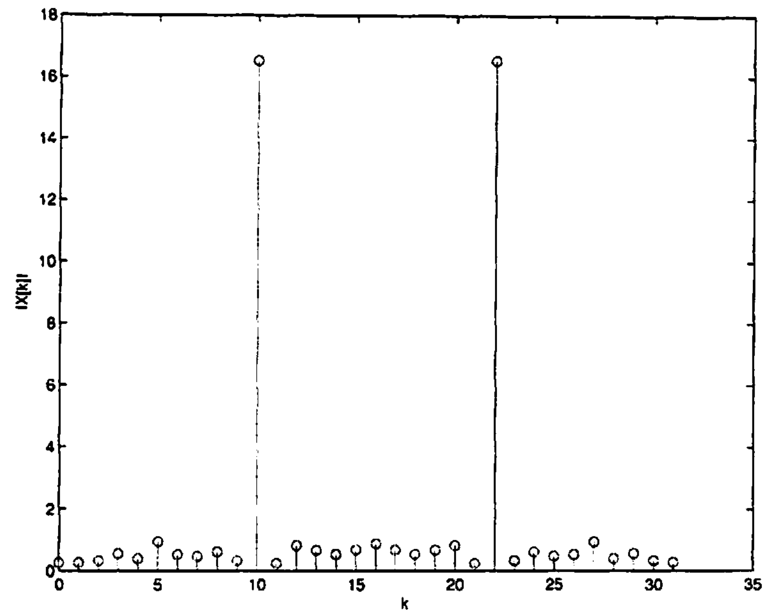
$$= \pi T\,\delta(\omega - \omega_0 + k2\pi) + \pi T\,\delta(\omega + \omega_0 + k2\pi)$$ (scaling property of dirac δ-function)

$$\Rightarrow X(\omega) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + k2\pi) + \pi \sum_{k=-\infty}^{\infty} \delta(\omega + \omega_0 + k2\pi)$$ ✓

2. 20 points. Consider the system shown below.



Suppose you plot the magnitude of the DFT, i.e. plot $|X[k]|$ versus k, with the Matlab command `stem(0:31,abs(fft(x)))` and get the result shown below. What is/are the strong frequency/frequencies present in the original continuous time signal $x(t)$? Explain.



If we assume there was no aliasing, then we can convert $k \to \omega_0 \to \Omega_0$ as follows:

DFT index $k \to$ normalized frequency $\omega_0 = \frac{2\pi k}{N}$

$\to$ non-normalized frequency $\Omega_0 = \omega_0 \cdot F_T = \frac{2\pi k F_T}{N}$

We see a strong signal at $k = 10 \to \omega_0 = \frac{2\pi \cdot 10}{32}$

Hence $\Omega_0 = \frac{2\pi \cdot 10 \cdot 16000}{32} = 2\pi \cdot 5000 \to 5\text{kHz}$

same

We see another strong signal at $k = 22$. But this is just a spectral replica of our 5KHz tone. Check:

$$2\pi - \frac{2\pi \cdot 22}{32} = \frac{32}{32} \cdot 2\pi - \frac{22}{32} \cdot 2\pi = \frac{10}{32} \cdot 2\pi$$

So the only frequency (strongly) present in the signal is 5kHz.

3. 20 points. Suppose you have a discrete-time system described by the difference equation

$$y[n] = px[n] + qy[n-1] + ry[n-2]$$ ← causal

where p, q, r are all known constants. Write the transfer function for this system.

$$Y(z)\left[1 - qz^{-1} - rz^{-2}\right] = p\,X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{p}{1 - qz^{-1} - rz^{-2}}$$

ROC extending outward from largest pole.

We can compute the poles via the quadratic eq:

poles at : $\dfrac{q \pm \sqrt{q^2 + 4r}}{2}$