ECE503 Midterm Exam

Your Name: SOLUTION Your box #: ____________

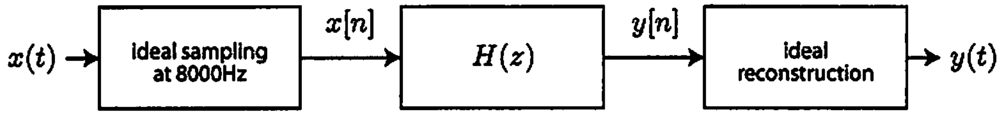
February 27, 2012

Tips:

- Look over all of the questions before starting.
- Budget your time to allow yourself enough time to work on each question.
- Write neatly and show your work! Points will be deducted for a disorderly presentation of your solution.
- This exam is worth a total of 300 points.
- This exam is to be completed in 90 minutes.
- You are permitted to consult your textbook, one handwritten "cheat sheet", and a calculator.
- Attach your "cheat sheet" to the exam when you hand it in.

problem 1	problem 2	problem 3	problem 4	total midterm exam score
80 points	72 points	70 points	78 points	300 points

1. 80 points. Consider the digital signal processing system shown below.



Suppose $x(t) = 3\cos(2\pi \cdot 1000 \cdot t) + 2\sin(2\pi \cdot 60 \cdot t)$ for all $t \in \mathbb{R}$. Design a linear time invariant system $H(z)$ such that the output $y(t) = \cos(2\pi \cdot 1000 \cdot t + \phi)$ and explicitly determine ϕ.

Let $h[n] = \alpha_0 \delta[n] + \alpha_1 \delta[n-1] + \alpha_0 \delta[n-2]$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n} = \alpha_0 + \alpha_1 e^{-j\omega} + \alpha_0 e^{-j2\omega}$$

$$= e^{-j\omega}\left[\alpha_1 + \alpha_0\left(e^{j\omega} + e^{-j\omega}\right)\right]$$

$$= e^{-j\omega}\left[\alpha_1 + 2\alpha_0 \cos\omega\right]$$

$|H(\omega)| = |\alpha_1 + 2\alpha_0 \cos\omega|$

Let $\omega_0 = \frac{2\pi 60}{8000}$ (normalized frequency)

$\omega_1 = \frac{2\pi \cdot 1000}{8000}$ (")

We want $|H(\omega_0)| = 0$ and $|H(\omega_1)| = \frac{1}{3}$

i) $\alpha_1 + 2\alpha_0 \cos\omega_0 = 0$

ii) $\alpha_1 + 2\alpha_0 \cos\omega_1 = 1/3$

solve for α_0 and α_1

$$2\alpha_0\left(\cos(\omega_1) - \cos(\omega_0)\right) = \frac{1}{3} \Rightarrow \alpha_0 = -0.5712$$

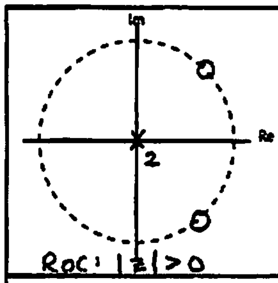
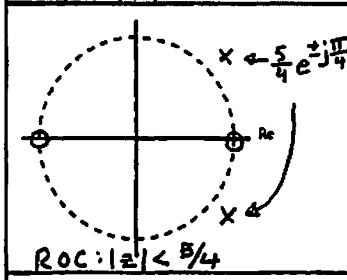
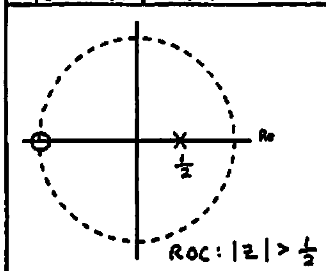
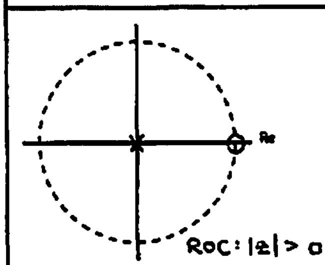
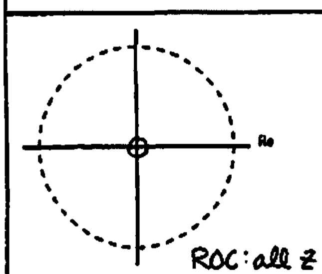
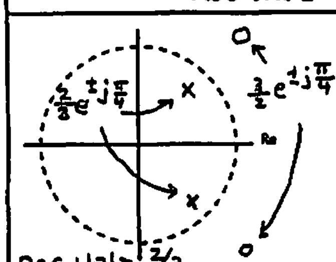
then $\alpha_1 = 1.1411$

Hence $h[n] = -0.5712\,\delta[n] + 1.1411\,\delta[n-1] - 0.5712\,\delta[n-2]$

and $\boxed{H(z) = -0.5712 + 1.1411 z^{-1} - 0.5712 z^{-2}}$ ROC: $|z| > 0$.

$\measuredangle H(\omega) = -\omega$ so $\measuredangle H(\omega_1) = -\omega_1 = \boxed{-\pi/4 = \phi}$

2. 72 points (12 points each). Classify the systems below based on their pole-zero plots.

$\frac{(z-e^{j\omega_0})(z-e^{-j\omega_0})}{z^2}$ $\frac{z^2-az+1}{z^2}$		○ lowpass ○ highpass ○ bandpass ● bandstop/notch ○ allpass ○ none of the above	● FIR type 1 ○ FIR type 2 ○ FIR type 3 ○ FIR type 4 ○ FIR no type ○ IIR ○ none of the above	● causal ○ not causal ○ inconclusive ● stable ○ not stable ○ inconclusive
		○ lowpass ○ highpass ● bandpass ○ bandstop/notch ○ allpass ○ none of the above	○ FIR type 1 ○ FIR type 2 ○ FIR type 3 ○ FIR type 4 ○ FIR no type ● IIR ○ none of the above	○ causal ● not causal ○ inconclusive ● stable ○ not stable ○ inconclusive
		● lowpass ○ highpass ○ bandpass ○ bandstop/notch ○ allpass ○ none of the above	○ FIR type 1 ○ FIR type 2 ○ FIR type 3 ○ FIR type 4 ○ FIR no type ● IIR ○ none of the above	● causal ○ not causal ○ inconclusive ● stable ○ not stable ○ inconclusive
$\frac{z-1}{z} = 1 - z^{-1}$		○ lowpass ● highpass ○ bandpass ○ bandstop/notch ○ allpass ○ none of the above	○ FIR type 1 ○ FIR type 2 ○ FIR type 3 ● FIR type 4 ○ FIR no type ○ IIR ○ none of the above	● causal ○ not causal ○ inconclusive ● stable ○ not stable ○ inconclusive
[0, 1, 0] $h[n] = \delta[n+1]$ $H(z)=z$ $H(\omega)=e^{j\omega}$ $\|H(\omega)\|=1$		○ lowpass ○ highpass ○ bandpass ○ bandstop/notch ● allpass ○ none of the above	● FIR type 1 ○ FIR type 2 ○ FIR type 3 ○ FIR type 4 ● FIR no type ○ IIR ○ none of the above	○ causal ● not causal ○ inconclusive ● stable ○ not stable ○ inconclusive
		○ lowpass ○ highpass ○ bandpass ○ bandstop/notch ● allpass ○ none of the above	○ FIR type 1 ○ FIR type 2 ○ FIR type 3 ○ FIR type 4 ○ FIR no type ● IIR ○ none of the above	● causal ○ not causal ○ inconclusive ● stable ○ not stable ○ inconclusive

3. 70 points. Suppose $x[n]$ is a length-N sequence defined for $n = 0, 1, \ldots, N-1$ with N even and

$$y[n] = (-1)^n x[n]$$

is another length-N sequence. Relate $Y[k] = \mathsf{DFT}\{y[n]\}$ and $X[k] = \mathsf{DFT}\{x[n]\}$.

Modulation theorem (p. 230)

$$y[n] = g[n]\,x[n] \longleftrightarrow \frac{1}{N}\sum_{\ell=0}^{N-1} G[\ell]\,X[\langle k-\ell\rangle_N]$$

where, here, $g[n] = (-1)^n$

Let's compute $G[\ell] = \sum_{n=0}^{N-1} g[n]\,W_N^{\ell n} = \sum_{n=0}^{N-1} (-1)^n W_N^{\ell n}$

Note $G[0] = \sum_{n=0}^{N-1} (-1)^n = 0$ (N is even)

$$G[1] = \sum_{n=0}^{N-1} (-1)^n W_N^{n} = 0$$

$$\vdots$$

$$G\left[\tfrac{N}{2}\right] = \sum_{n=0}^{N-1} (-1)^n W_N^{\frac{N}{2}n} = \sum_{n=0}^{N-1} (-1)^n e^{-j\pi n}$$

$$= \sum_{n=0}^{N-1} (-1)^n (-1)^n = N$$

All other $G[\ell] = 0$ except $G\left[\tfrac{N}{2}\right]$

So $y[n] = g[n]\,x[n] \longleftrightarrow \frac{1}{N}\sum_{\ell=0}^{N-1} N\delta\left[\ell - \tfrac{N}{2}\right] X[\langle k-\ell\rangle_N]$

$$\longleftrightarrow X\left[\langle k - \tfrac{N}{2}\rangle_N\right]$$

Hence $Y[k] = X\left[\langle k - \tfrac{N}{2}\rangle_N\right]$

or

$$\boxed{Y[k] = \begin{cases} X\left[k - \frac{N}{2}\right] & , \ k = \frac{N}{2}, \frac{N}{2}+1, \ldots, N-1 \\ X\left[k + N - 1 - \frac{N}{2}\right] & \ k = 0, 1, \ldots, \frac{N}{2}-1 \end{cases}}$$

4. 78 points total. Suppose you have a linear time invariant system with impulse response

$$h[n] = \delta[n] - 2.5\delta[n-1] + \delta[n-2]$$

(a) 10 points. Is this a minimum, maximum, or mixed phase system? Explain why.

$$H(z) = 1 - 2.5z^{-1} + z^{-2} = (1 - 0.5z^{-1})(1 - 2z^{-1}) \quad \text{RoC}: |z| > 0$$

zeros inside and outside unit circle

$\Rightarrow \boxed{\text{mixed phase}}$

(b) 18 points. Prove this system has linear phase and compute its group delay.

$$H(\omega) = 1 - 2.5e^{-j\omega} + e^{-j2\omega}$$

$$= e^{-j\omega}\left[-2.5 + e^{j\omega} + e^{-j\omega}\right]$$

$$= e^{-j\omega}\left[-2.5 + 2\cos(\omega)\right]$$

$$|H(\omega)| = |-2.5 + 2\cos(\omega)|$$

$$\measuredangle H(\omega) = -\omega = \theta(\omega)$$

Hence this system has linear phase.

The group delay is simply $\boxed{\tau_g(\omega) = \frac{-d\theta(\omega)}{d\omega} = 1 \text{ Sample}}$

(c) 50 points. Find a causal delay complementary system also with linear phase and compute its impulse response.

To find a delay complementary system with linear phase, we set

$$G(z) + H(z) = z^{-1}$$

$$\Rightarrow G(z) = z^{-1} - H(z)$$

$$= z^{-1} - 1 + 2.5z^{-1} - z^{-2}$$

$$= -1 + 3.5z^{-1} - z^{-2}$$

Note the impulse response is symmetric, hence this system has linear phase. In fact

$$G(\omega) = -1 + 3.5e^{-j\omega} - e^{-j2\omega}$$

$$= e^{-j\omega}\left[3.5 - \left[e^{j\omega} + e^{-j\omega}\right]\right]$$

$$= e^{-j\omega}\left[3.5 - 2\cos(\omega)\right]$$

$\measuredangle G(\omega) = -\omega$ so linear phase.

Impulse response is then $\boxed{g[n] = -\delta[n] + 3.5\,\delta[n-1] - \delta[n-2]}$

Easy to check $g[n] + h[n] = \delta[n-1] \Rightarrow$ delay complementary