ECE503 Homework Assignment Number 7

Due by 8:50pm on Monday 26-Mar-2012

IMPORTANT: Please place your ECE mailbox number on all homework assignments. Your ECE mailbox number can be found on the course web page.

Make sure your reasoning and work are clear to receive full credit for each problem. Points will be deducted for a disorderly presentation of your solution. Please also refer to the course academic honesty policies regarding collaboration on homework assignments.

1. 3 points. Suppose a discrete-time signal

$$x[n] = \begin{cases} 0.9^n & n = 0, \dots, 9 \\ 0 & \text{otherwise.} \end{cases}$$

is sent through an ideal reconstruction filter with sampling period $T = \frac{1}{10}$ seconds to generate a continuous-time signal $x(t)$.

   (a) Note that $x[n] = 0$ for all $n < 0$. Does $x(t) = 0$ for all $t < 0$? Why or why not?

   (b) Determine the value of $x(t)$ at time $t = 0.5$ seconds.

   (c) Now suppose the sampling period $T = \frac{1}{5}$ seconds. Determine the value of $x(t)$ at time $t = 0.5$ seconds.

2. 4 points. Suppose you have a real-valued continuous-time signal $x(t) = \cos(2\pi \cdot 10 \cdot t)$ and this signal is ideally sampled at frequency $F_T = 30$ Hertz to to generate a discrete-time signal $x[n]$.

   (a) Sketch the magnitude of $X(\Omega)$ and the magnitude of $X(\omega)$, explicitly showing any periodicity in the spectra. Is there aliasing?

   (b) Now suppose $x[n]$ is upsampled by a factor of two, resulting in $y[n]$. Sketch the magnitude of $Y(\omega)$, explicitly showing any periodicity in the spectrum.

   (c) Now suppose $x[n]$ is downsampled by a factor of two, resulting in $z[n]$. Sketch the magnitude of $Z(\omega)$, explicitly showing any periodicity in the spectrum.

   (d) Now suppose $z[n]$ is sent to an ideal reconstruction filter to generate $z(t)$. Note this ideal reconstruction filter will use a period of $T = \frac{1}{15}$ seconds because the sampling rate of $z[n]$ is 15 Hertz. Can you find a closed-form expression for $z(t)$?

3. 4 points total. Suppose you wish to design a "bandpass" filter that has unity magnitude at $\omega_1 = \frac{\pi}{4}$ and has magnitude $\frac{1}{\sqrt{2}}$ at $\omega_1 = \frac{\pi}{4} \pm \omega_0$.

   (a) 2 points. Design a filter that meets these specifications when $\omega_0 = \pi/8$. Use Matlab to plot the magnitude response and confirm it agrees with the specifications. Is this a good bandpass filter? Why or why not?

(b) 2 points. Discuss what happens as ω_0 gets small.

4. 3 points. Suppose $x[n]$ is a length-N sequence and $y[n]$ is a length-$2N$ sequence formed by repeating $x[n]$ twice, i.e.

$$y[n] = \begin{cases} x[n] & n = 0, 1, \ldots, N-1 \\ x[n-N] & n = N, \ldots, 2N-1. \end{cases}$$

Let $Y[k]$ be the $2N$-point DFT of $y[n]$ for $k = 0, \ldots, 2N-1$ and let $Z[k] = Y[2k]$ for $k = 0, 1, \ldots, N-1$. In other words, $Z[k]$ is a down sampled version of $Y[k]$, where the downsampling in occurring in the frequency domain. Relate $z[n] = \mathsf{IDFT}\{Z[k]\}$ to $x[n]$ for $n = 0, \ldots, N-1$.

5. 4 points. Mitra 4.43. Suggested approach: Use z-domain analysis to find the zero-state response. To find the zero-input response, find the homogeneous solution and apply the initial conditions to solve for the unknown constants. Compute the total response as the sum of the zero-state response and the zero-input response. You can check your answer with Matlab.

6. 3 points. Compute the impulse response of the system in Mitra 4.43.

7. 4 points. Mitra 5.9.