1. 3 points. Suppose a discrete-time signal
\[ x[n] = \begin{cases} 
0.9^n & n = 0, \ldots, 9 \\
0 & \text{otherwise}
\end{cases} \]
is sent through an ideal reconstruction filter with sampling period \( T = \frac{1}{10} \) seconds to generate a continuous-time signal \( x(t) \).

(a) Note that \( x[n] = 0 \) for all \( n < 0 \). Does \( x(t) = 0 \) for all \( t < 0 \)? Why or why not?

(b) Determine the value of \( x(t) \) at time \( t = 0.5 \) seconds.

(c) Now suppose the sampling period \( T = \frac{1}{15} \) seconds. Determine the value of \( x(t) \) at time \( t = 0.5 \) seconds.

2. 4 points. Suppose you have a real-valued continuous-time signal \( x(t) = \cos(2\pi \cdot 10 \cdot t) \) and this signal is ideally sampled at frequency \( F_T = 30 \) Hertz to generate a discrete-time signal \( x[n] \).

(a) Sketch the magnitude of \( X(\Omega) \) and the magnitude of \( X(\omega) \), explicitly showing any periodicity in the spectra. Is there aliasing?

(b) Now suppose \( x[n] \) is upsampled by a factor of two, resulting in \( y[n] \). Sketch the magnitude of \( Y(\omega) \), explicitly showing any periodicity in the spectrum.

(c) Now suppose \( x[n] \) is downsampled by a factor of two, resulting in \( z[n] \). Sketch the magnitude of \( Z(\omega) \), explicitly showing any periodicity in the spectrum.

(d) Now suppose \( z[n] \) is sent to an ideal reconstruction filter to generate \( z(t) \). Note this ideal reconstruction filter will use a period of \( T = \frac{1}{15} \) seconds because the sampling rate of \( z[n] \) is 15 Hertz. Can you find a closed-form expression for \( z(t) \)?

3. 4 points total. Suppose you wish to design a “bandpass” filter that has unity magnitude at \( \omega_1 = \frac{\pi}{4} \) and has magnitude \( \frac{1}{\sqrt{2}} \) at \( \omega_1 = \frac{\pi}{4} \pm \omega_0 \).

(a) 2 points. Design a filter that meets these specifications when \( \omega_0 = \pi/8 \). Use Matlab to plot the magnitude response and confirm it agrees with the specifications. Is this a good bandpass filter? Why or why not?
(b) 2 points. Discuss what happens as $\omega_0$ gets small.

4. 3 points. Suppose $x[n]$ is a length-$N$ sequence and $y[n]$ is a length-$2N$ sequence formed by repeating $x[n]$ twice, i.e.

$$
y[n] = \begin{cases} 
x[n] & n = 0, 1, \ldots, N - 1 \\
x[n - N] & n = N, \ldots, 2N - 1.
\end{cases}
$$

Let $Y[k]$ be the $2N$-point DFT of $y[n]$ for $k = 0, \ldots, 2N - 1$ and let $Z[k] = Y[2k]$ for $k = 0, 1, \ldots, N - 1$. In other words, $Z[k]$ is a down sampled version of $Y[k]$, where the downsampling is occurring in the frequency domain. Relate $z[n] = \text{IDFT}\{Z[k]\}$ to $x[n]$ for $n = 0, \ldots, N - 1$.

5. 4 points. Mitra 4.43. Suggested approach: Use $z$-domain analysis to find the zero-state response. To find the zero-input response, find the homogeneous solution and apply the initial conditions to solve for the unknown constants. Compute the total response as the sum of the zero-state response and the zero-input response. You can check your answer with Matlab.

6. 3 points. Compute the impulse response of the system in Mitra 4.43.

7. 4 points. Mitra 5.9.