Example 2 in Lecture 4 (corrected)

$$x[n] = \left(-\tfrac{1}{3}\right)^n \mu[n] - \left(\tfrac{1}{2}\right)^n \mu[-n-1]$$

$$= x_1[n] - x_2[n]$$

We know $x_1[n] \longleftrightarrow X_1(z) = \dfrac{1}{1+\frac{1}{3}z^{-1}}$ ROC: $|z| > \frac{1}{3}$

Lets compute $X_2(z)$ via the definition

$$X_2(z) = \sum_{n=-\infty}^{-1} \left(\tfrac{1}{2}\right)^n z^{-n} = \sum_{n=1}^{\infty} (2z)^n = 2z + (2z)^2 + (2z)^3 + \cdots$$

$$(1-2z)X_2(z) = \left(2z + (2z)^2 + \ldots\right) - \left((2z)^2 + (2z)^3 + \ldots\right) = 2z$$

$$\Rightarrow X_2(z) = \frac{2z}{1-2z} = \frac{1}{\frac{1}{2z}-1} = -\frac{1}{1-\frac{1}{2}z^{-1}} \qquad \text{ROC: } |z| < \tfrac{1}{2}$$

This is the − sign I dropped in lecture.

Hence $X(z) = X_1(z) + X_2(z)$ ROC: $\frac{1}{3} < |z| < \frac{1}{2}$

$$= \frac{1}{1+\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} = \frac{2z\left(z-\frac{1}{12}\right)}{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{2}\right)}$$