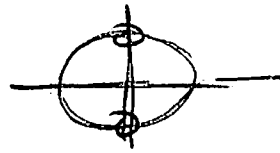


7.76 a)

Find delay complementary TF

$$H_1(z) = \frac{1}{4} (1 + z^{-2})^2$$



$$= \frac{1}{4} (1 + 2z^{-2} + z^{-4})$$

$$H_1(\omega) = \frac{1}{4} (1 + 2e^{-j2\omega} + e^{-j4\omega})$$

$$= \frac{1}{4} e^{-j2\omega} (2 + e^{+j2\omega} + e^{-j2\omega})$$

$$= \underbrace{e^{-j2\omega}}_{\substack{\uparrow \\ \text{linear} \\ \text{phase} \\ \text{response}}} \underbrace{\frac{1}{2} (1 + \cos(2\omega))}_{\substack{\uparrow \\ \text{amplitude response} \\ \text{(real)}}} H_1(\omega)$$

$e^{-j\omega(N/2)}$ ← group delay $N/2 = 2$

(7.117) says we form the delay complementary filter with same group delay as $H_1(z)$

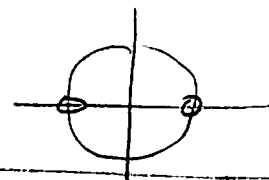
$$H_2(z) = z^{-N/2} - H_1(z)$$

Here we have $H_2(z) = z^{-2} - \frac{1}{4} (1 + z^{-2})^2$

$$= z^{-2} - \frac{1}{4} - \frac{1}{2} z^{-2} - \frac{1}{4} z^{-4}$$

$$= -\frac{1}{4} + \frac{1}{2} z^{-2} - \frac{1}{4} z^{-4}$$

$$= -\frac{1}{4} (1 - z^{-2})^2$$



What if we did $H_2(z) = 1 - H_1(z)$ (set $n_0 = 0$)

$$\text{then } H_2(z) = \frac{3}{4} - \frac{1}{2} z^{-2} + \frac{1}{4} z^{-4}$$

★ NOT LINEAR PHASE ★

To get $H_2(z)$ to also be linear phase, we want the

delay complementary filter to have the same group delay as $H_1(\omega)$

$$\Rightarrow H_2(\omega) = e^{-j2\omega} (1 - H_1(\omega)) = e^{-j2\omega} \left(-\frac{1}{2} - \frac{1}{2} \cos(2\omega) \right)$$

$$H_2(z) = z^{-2} \left(-\frac{1}{2} - \frac{1}{4} z^{-2} - \frac{1}{4} z^2 \right) = -\frac{1}{4} (2z^{-2} + z^2 + 1)$$

Deconvolution example.

7.92c)

$$h[0] = \frac{y[0]}{x[0]}$$

$$\dots \frac{x[2] \ x[1] \ x[0]}{h[0] \ h[1] \ h[2] \ \dots} = \frac{y[0]}{y[0]}$$

$$h[0] = \frac{12}{4} = 3$$

$$h[1] = \frac{y[1] - h[0]x[1]}{x[0]}$$

$$\dots \frac{\cancel{x[1]} \ \cancel{x[0]}}{\cancel{x[0]} \ h[0] \ h[1] \ \dots} = \frac{y[1]}{y[1]}$$

$$= \frac{-13 - 3}{4} = \frac{-16}{4} = -4$$

$$h[1] = -4$$

$$\dots \frac{\cancel{x[2]} \ \cancel{x[1]} \ \cancel{x[0]}}{\cancel{x[0]} \ h[0] \ h[1] \ h[2] \ \dots} = \frac{y[2]}{y[2]}$$

$$h[2] = \frac{y[2] - h[0]x[2] - h[1]x[1]}{x[0]}$$

$$= \frac{-5 - 3(-3) - (-4)(1)}{4} = \frac{8}{4}$$

$$h[2] = 2$$

done (check in matlab)