Solutions to Mitra 8.53 and 8.54

8.53 (a) $H_4(z) = 2 + 23z^{-1} + 73z^{-2} + 43z^{-3} - 15z^{-4}$,

$G_4(z) = -4 - 24z^{-1} + 85z^{-2} + 2z^{-3} - 3z^{-4}$.

$\delta_4 = \frac{-15}{-3} = 5,\ \gamma_4 = \frac{-4}{2} = -2,\ \ K_4 = \frac{1}{1+10} = \frac{1}{11}$.

$H_3(z) = K_4\big(H_4(z) - \delta_4 G_4(z)\big) = 2 + 13z^{-1} - 32z^{-2} + 3z^{-3}$,

$G_3(z) = K_4\big(G_4(z) - \gamma_4 H_4(z)\big) = 2 + 21z^{-1} + 8z^{-2} - 3z^{-3}$.

$\delta_3 = \frac{3}{-3} = -1,\ \gamma_3 = \frac{2}{2} = 1,\ \ K_3 = \frac{1}{1+1} = 0.5$.

$H_2(z) = K_3\big(H_3(z) - \delta_3 G_3(z)\big) = 2 + 17z^{-1} - 12z^{-2}$,

$G_2(z) = K_3\big(G_3(z) - \gamma_3 H_3(z)\big) = 4 + 20z^{-1} - 3z^{-2}$.

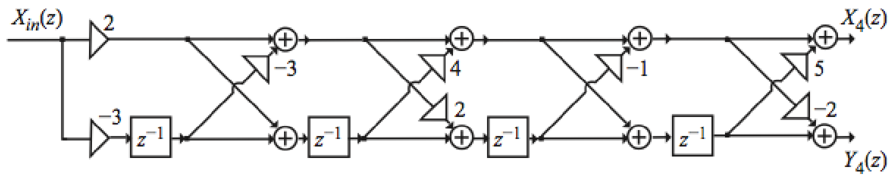
$\delta_2 = \frac{-12}{-3} = 4,\ \gamma_2 = \frac{4}{2} = 2,\ \ K_2 = \frac{1}{1-8} = -\frac{1}{7}$.

$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 2 + 9z^{-1},\ G_1(z) = K_2\big(G_2(z) - \gamma_2 H_2(z)\big) = 2 - 3z^{-1}$.

$\delta_1 = \frac{9}{-3} = -3,\ \gamma_1 = \frac{2}{2} = 1,\ \ K_1 = \frac{1}{1+3} = 0.25$.

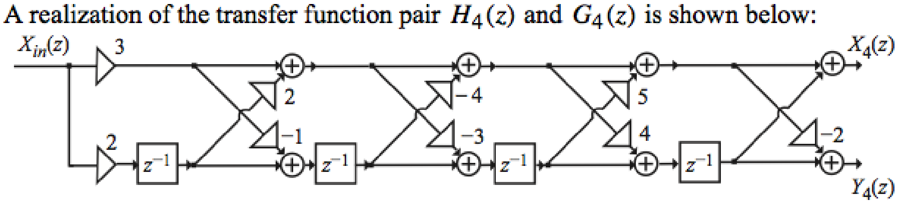
$\delta_0 = H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = 2,\ \gamma_0 = G_0(z) = K_1\big(G_1(z) - \gamma_1 H_1(z)\big) = -3$.

A realization of the transfer function pair $H_4(z)$ and $G_4(z)$ is shown below:



(b) $H_4(z) = 3 - 17z^{-1} - 28z^{-2} - 37z^{-3} + 2z^{-4}$,
$G_3(z) = 6 - 46z^{-1} - 111z^{-2} - 27z^{-3} + 2z^{-4}$.
$\delta_4 = \dfrac{2}{2} = 1,\ \gamma_4 = \dfrac{6}{3} = 2,\ \ K_4 = \dfrac{1}{1-2} = -1.$
$H_3(z) = K_4\big(H_4(z) - \delta_4 G_4(z)\big) = 3 - 29z^{-1} - 83z^{-2} + 10z^{-3}$,
$G_3(z) = K_4\big(G_4(z) - \gamma_4 H_4(z)\big) = 12 + 55z^{-1} - 47z^{-2} + 2z^{-3}$.
$\delta_3 = \dfrac{10}{2} = 5,\ \gamma_3 = \dfrac{12}{3} = 4,\ \ K_3 = \dfrac{1}{1-20} = -\dfrac{1}{19}.$
$H_2(z) = K_3\big(H_3(z) - \delta_3 G_3(z)\big) = 3 + 16z^{-1} - 8z^{-2}$,
$G_2(z) = K_3\big(G_3(z) - \gamma_3 H_3(z)\big) = -9 - 15z^{-1} + 2z^{-2}$.
$\delta_2 = \dfrac{-8}{2} = -4,\ \gamma_2 = \dfrac{-9}{3} = -3,\ \ K_2 = \dfrac{1}{1-12} = -\dfrac{1}{11}.$
$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 3 + 4z^{-1}$, $G_1(z) = K_2\big(G_2(z) - \gamma_2 H_2(z)\big) = -3 + 2z^{-1}$.
$\delta_1 = \dfrac{4}{2} = 2,\ \gamma_1 = \dfrac{-3}{3} = -1,\ \ K_1 = \dfrac{1}{1+2} = \dfrac{1}{3}.$
$\delta_0 = H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = 3,\ \gamma_0 = G_0(z) = K_1\big(G_1(z) - \gamma_1 H_1(z)\big) = 2.$

A realization of the transfer function pair $H_4(z)$ and $G_4(z)$ is shown below:



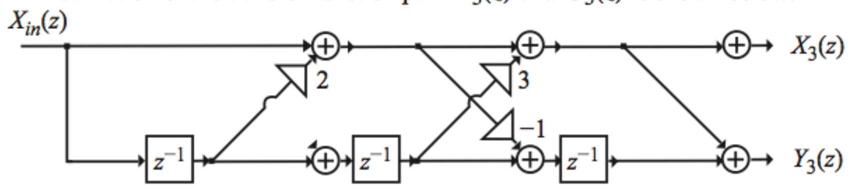
8.54 **(a)** $H_3(z) = 1 + z^{-1} + z^{-2} + z^{-3}$, $G_3(z) = 1 + 2z^{-1} + 3z^{-2}$.
Since $b_3^{(3)} = 0$, $\delta_3 \to \infty$ and it is thus a Case 1 situation. We re-label the transfer function pair as $H_3'(z) = 1 + 2z^{-1} + 3z^{-2}$, $G_3'(z) = 1 + z^{-1} + z^{-2} + z^{-3}$.
We now follow the realization method.
$\delta_3 = \dfrac{0}{1} = 0,\ \gamma_3 = \dfrac{1}{1} = 1,\ \ K_3 = \dfrac{1}{1-0} = 1.$
$H_2(z) = K_3\big(H_3(z) - \delta_3 G_3(z)\big) = K_3 H_3(z) = 1 + 2z^{-1} + 3z^{-2}$,
$G_2(z) = K_3\big(G_3(z) - \gamma_3 H_3(z)\big) = -1 - 2z^{-1} + z^{-2}$.
$\delta_2 = \dfrac{3}{1} = 3,\ \gamma_2 = \dfrac{-1}{1} = -1,\ \ K_2 = \dfrac{1}{1+3} = \dfrac{1}{4}.$

$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 1 + 2z^{-1}, \;\; G_1(z) = K_2\big(G_2(z) - \gamma_2 H_2(z)\big) = z^{-1}.$

$\delta_1 = \frac{2}{1} = 2, \; \gamma_1 = \frac{0}{1} = 0, \;\; K_1 = \frac{1}{1} = 1.$

$\delta_0 = H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = 1, \;\; \gamma_0 = G_0(z) = K_1\big(G_1(z) - \gamma_1 H_1(z)\big) = 1.$

A realization of the transfer function pair $H_3(z)$ and $G_3(z)$ is shown below:



(b) $H_3(z) = 2z^{-1} + 3z^{-2} + 4z^{-3}, \;\; G_3(z) = 1 + z^{-1} + z^{-2} + z^{-3}.$

Since $a_3^{(3)} = 0, \; \gamma_3 \to \infty$ and it is thus a Case 2 situation. We re-label the transfer function pair as $H_3'(z) = 1 + z^{-1} + z^{-2} + z^{-3}, \;\; G_3'(z) = 2z^{-1} + 3z^{-2} + 4z^{-3}.$

We now follow the realization method.

$\delta_3 = \frac{1}{4}, \; \gamma_3 = \frac{0}{1} = 0, \;\; K_3 = \frac{1}{1-0} = 1.$

$H_2(z) = K_3\big(H_3(z) - \delta_3 G_3(z)\big) = K_3 H_3(z) = 1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2},$

$G_2(z) = K_3\big(G_3(z) - \gamma_3 H_3(z)\big) = 2 + 3z^{-1} + 4z^{-2}.$

$\delta_2 = \frac{1/4}{4} = \frac{1}{16}, \; \gamma_2 = \frac{2}{1} = 2, \;\; K_2 = \frac{1}{1 - \frac{1}{8}} = \frac{8}{7}.$

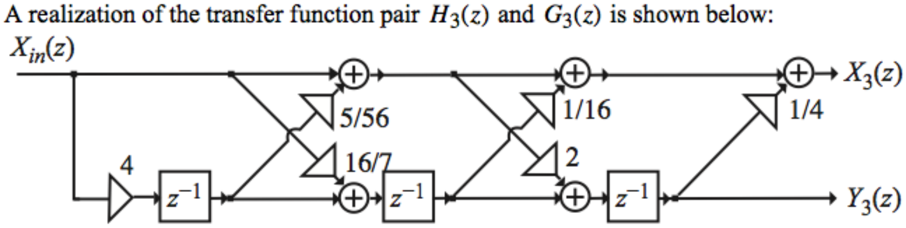
$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 1 + \frac{5}{14}z^{-1},$

$G_1(z) = K_2\big(G_2(z) - \gamma_2 H_2(z)\big) = \frac{16}{7} + 4z^{-1}.$

$\delta_1 = \frac{5/14}{4} = \frac{5}{56}, \; \gamma_1 = \frac{16/7}{1} = \frac{16}{7}, \;\; K_1 = \frac{1}{1 - \frac{5}{56}\cdot\frac{16}{7}} = \frac{49}{39}.$

$\delta_0 = H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = 1, \;\; \gamma_0 = G_0(z) = K_1\big(G_1(z) - \gamma_1 H_1(z)\big) = 4.$

A realization of the transfer function pair $H_3(z)$ and $G_3(z)$ is shown below:



8.54 **(c)** $H_3(z) = z^{-1} + z^{-2} + z^{-3}$, $G_3(z) = 1 + 2z^{-1} + 3z^{-2}$. Here $a_0^{(3)} = 0$ and $b_3^{(3)} = 0$. Hence, it is Case 3. We rewrite the above transfer functions as

$H_3(z) = z^{-1} H_2(z)$, $G_3(z) = G_2(z)$, where

$H_2(z) = 1 + z^{-1} + z^{-2}$, $G_2(z) = 1 + 2z^{-1} + 3z^{-2}$, and proceed with the realization.

$\delta_2 = \frac{1}{3}$, $\gamma_2 = \frac{1}{1} = 1$, $K_2 = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$.

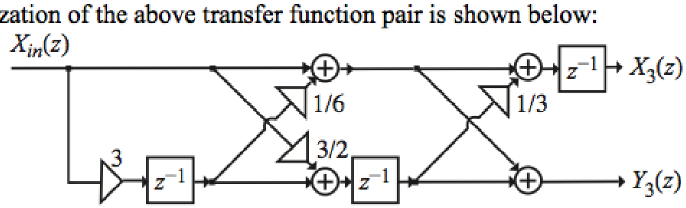
$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 1 + \frac{1}{2} z^{-1}$ and

$G_1(z) = K_2 z\big(G_2(z) - \gamma_2 H_2(z)\big) = \frac{3}{2} + 3z^{-1}$.

$\delta_1 = \frac{1}{6}$, $\gamma_1 = \frac{3}{2}$, $K_1 = \frac{1}{1 - \frac{1}{6} \cdot \frac{3}{2}} = \frac{4}{3}$.

$H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = \delta_0 = 1$, $G_0(z) = K_1 z\big(G_1(z) - \gamma_1 H_1(z)\big) = \gamma_0 = 3$.

The realization of the above transfer function pair is shown below:



(d) $H_3(z) = 1 + z^{-1} + z^{-2} + z^{-3}$, $G_3(z) = 4 + 2z^{-1} + 3z^{-2} + 4z^{-3}$. Here $a_3^{(3)} b_0^{(3)} = b_3^{(3)} a_0^{(3)}$. Hence, it is Case 4. We rewrite the above transfer functions as $H_3'(z) = H_3(z) + 1 = 2 + z^{-1} + z^{-2} + z^{-3}$, $G_3(z) = 4 + 2z^{-1} + 3z^{-2} + 4z^{-3}$, and proceed with the realization.

$\delta_3 = \frac{1}{4}$, $\gamma_3 = \frac{4}{2} = 2$, $K_3 = \frac{1}{1 - \frac{1}{2}} = 2$.

$H_2(z) = K_3\left(H_3'(z) - \delta_3 G_3(z)\right) = 2 + z^{-1} + \frac{1}{2} z^{-2}$,

$G_2(z) = K_3\left(G_3(z) - \gamma_3 H_3'(z)\right) = 2z^{-1} + 4z^{-2}$.

$\delta_2 = \frac{1/2}{2} = \frac{1}{4}$, $\gamma_2 = 0$, $K_2 = 1$.

$H_1(z) = K_2\big(H_2(z) - \delta_2 G_2(z)\big) = 2 + \frac{3}{4} z^{-1}$,

$G_1(z) = K_2\big(G_2(z) - \gamma_2 H_2(z)\big) = 2 + 4z^{-1}$.

$\delta_1 = \frac{3}{16}$, $\gamma_1 = 1$, $K_1 = \frac{1}{1 - \frac{3}{16}} = \frac{16}{13}$.

$H_0(z) = K_1\big(H_1(z) - \delta_1 G_1(z)\big) = \delta_0 = 2,\ G_0(z) = K_1 z\big(G_1(z) - \gamma_1 H_1(z)\big) = \gamma_0 = 4.$
The realization of the above transfer function pair is shown below: