Digital Signal Processing
Discrete-Time Processing of Continuous-Time Signals

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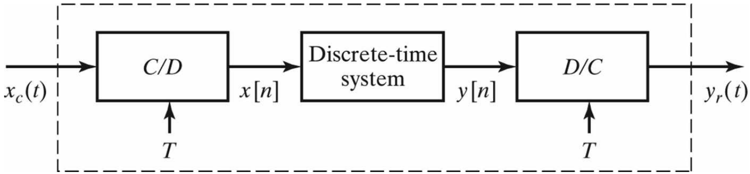
A common application of DT systems is to process CT signals:



Note that the boxed area is actually a continuous-time system.

Some questions:

- Under what conditions will the system from $x_c(t)$ to $y_r(t)$ be LTI?
- How can we design the DT system to achieve the desired overall response from $x_c(t)$ to $y_r(t)$?

An Example Real-Time DSP System for Audio Signals

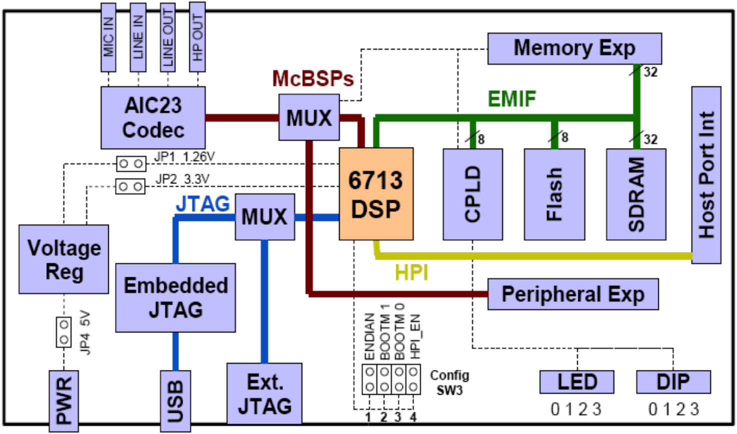
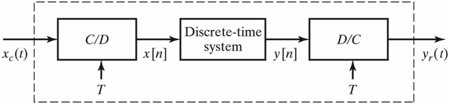


Figure 1-1, Block Diagram C6713 DSK

DT-LTI Processing: Signal Relations

Assume the DT system is LTI with impulse response $h[n]$ and freq. response $H(e^{j\omega})$.



We have previously shown:

$$
\begin{aligned}
x[n] &= x_c(nT) \\
X(e^{j\omega}) &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right) \\
y[n] &= h[n] * x[n] \\
Y(e^{j\omega}) &= H(e^{j\omega})X(e^{j\omega}) \\
y_r(t) &= \sum_{n=-\infty}^{\infty} y[n] \frac{\sin(\pi(t-nT)/T)}{\pi(t-nT)/T} \\
Y_r(j\Omega) &= H_r(j\Omega)Y(e^{j\Omega T}) = \begin{cases} TY(e^{j\Omega T}) & |\Omega| < \pi/T \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
$$

When is the Overall System LTI?

Using the prior frequency-domain results, we can write the output as

$$\begin{aligned} Y_r(j\Omega) &= H_r(j\Omega)Y(e^{j\Omega T}) \\ &= H_r(j\Omega)H(e^{j\Omega T})X(e^{j\Omega T}) \\ &= H_r(j\Omega)H(e^{j\Omega T})\frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right) \end{aligned}$$

Recall that $\Omega = \omega/T$ and

$$H_r(j\Omega) = \begin{cases} T & |\Omega| < \pi/T \\ 0 & \text{otherwise.} \end{cases}$$

Hence a sufficient condition for the overall system to be LTI is for there to be no aliasing, i.e. $X_c(j\Omega) = 0$ for all $|\Omega| \geq \pi/T$. This causes the reconstruction filter to remove all the images except at $k = 0$ so that

$$Y_r(j\Omega) = H(e^{j\Omega T})X_c(j\Omega)$$

Effective CT Filter

If no aliasing occurs (or if the DT filter removes any aliasing), we have

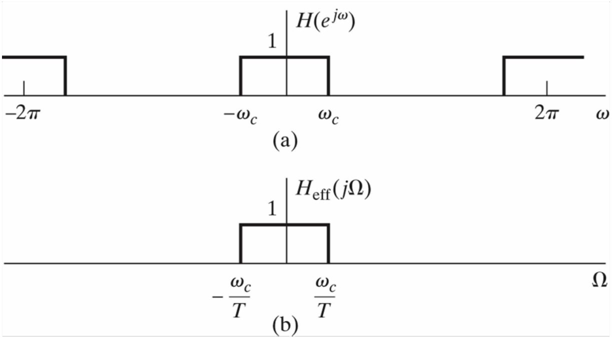
$$Y_r(j\Omega) = H(e^{j\Omega T})X_c(j\Omega)$$

The overall continuous-time system then effectively has the frequency response

$$H_{\text{eff}}(j\Omega) = \begin{cases} H(e^{j\omega})|_{\omega=\Omega T} & |\Omega| < \pi/T \\ 0 & \text{otherwise.} \end{cases}$$

Note that, under our assumption that $X_c(j\Omega) = 0$ for all $|\Omega| \geq \pi/T$, it doesn't matter what $H_{\text{eff}}(j\Omega)$ is for $|\Omega| \geq \pi/T$. Nevertheless, it is conventional to set $H_{\text{eff}}(j\Omega) = 0$ for $|\Omega| \geq \pi/T$.

Ideal Lowpass Filter Example



Remarks:

- The key relation here is $\Omega = \omega/T$.
- There are two ways to change the CT-system cutoff frequency $\Omega_c = \omega_c/T$. You can change the DT filter cutoff frequency ω_c and/or the sampling period T.