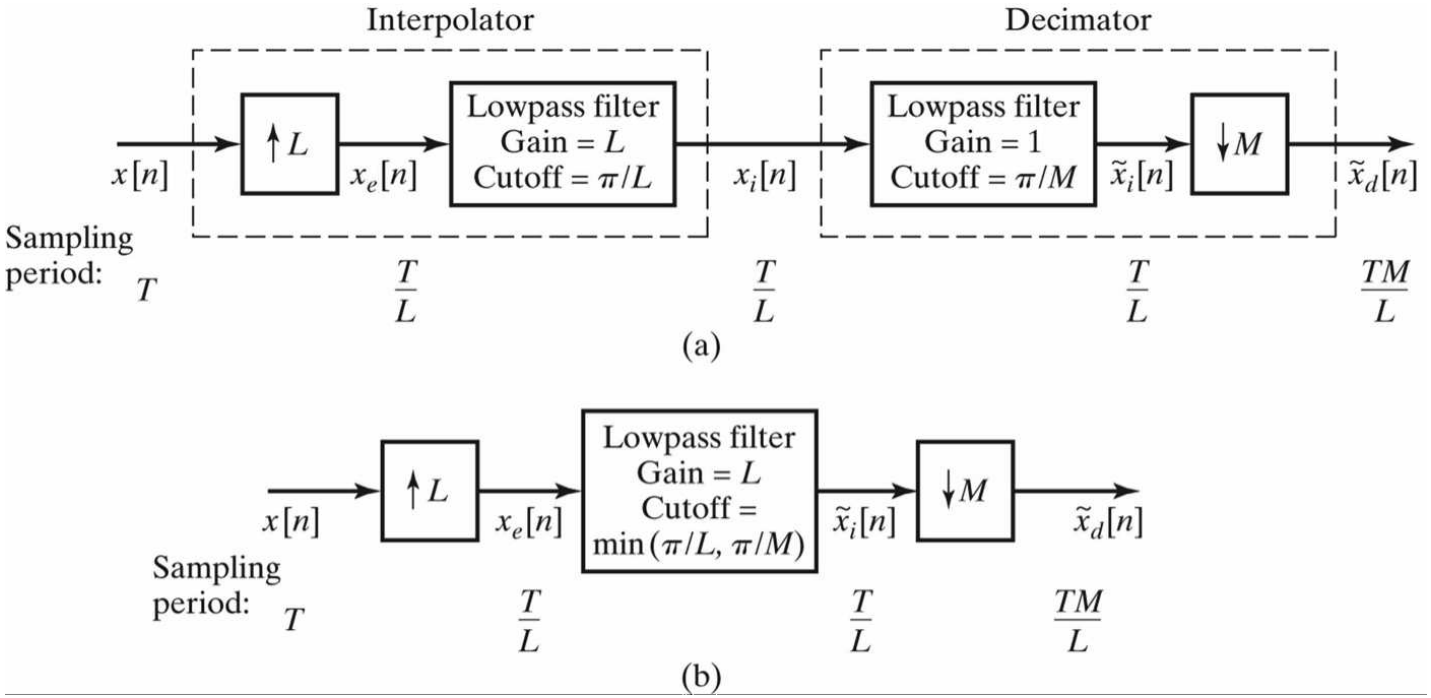
Digital Signal Processing
Efficient Sample Rate Conversion

D. Richard Brown III

Rational Sample Rate Conversion

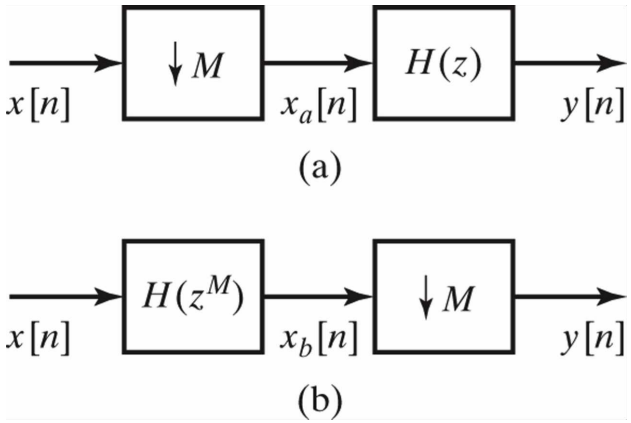


Example: Suppose we wish to change the sampling rate by a factor of $\frac{50}{49}$.

1. Upsample by factor of $L = 50$.
2. Lowpass filter with cutoff frequency $\frac{\pi}{50}$.
3. Downsample by factor of $M = 49$.

Exchanging the Order of Downsampling and Filtering

Equivalent structures:



Example $x[n] = \{\underline{1}, 2, 3, 4, 5, 6\}$, $H(z) = 10 + 20z^{-1}$, and $M = 2$.

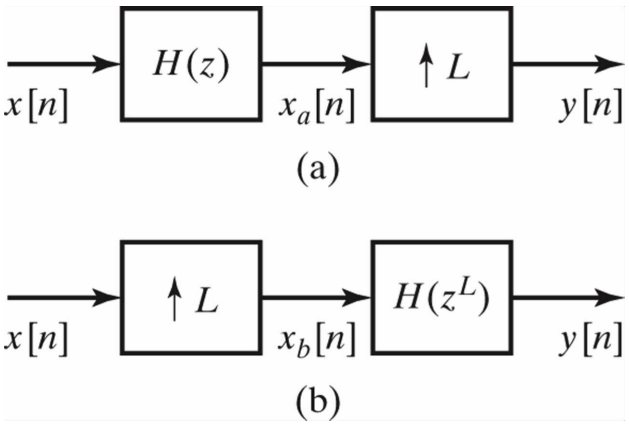
Approach (a): $x_a[n] = \{\underline{1}, 3, 5\}$ and $y[n] = \{\underline{10}, 50, 110, 100\}$.

Approach (b): $H(z^M) = 10 + 20z^{-2}$.
$x_b[n] = \{\underline{10}, 20, 50, 80, 110, 140, 100, 120\}$ and $y[n] = \{\underline{10}, 50, 110, 100\}$.

Which approach is easier to implement?

Exchanging the Order of Upsampling and Filtering

Equivalent structures:



Example $x[n] = \{\underline{1}, 2, 3, 4\}$, $H(z) = 10 + 20z^{-1}$, and $L = 2$.

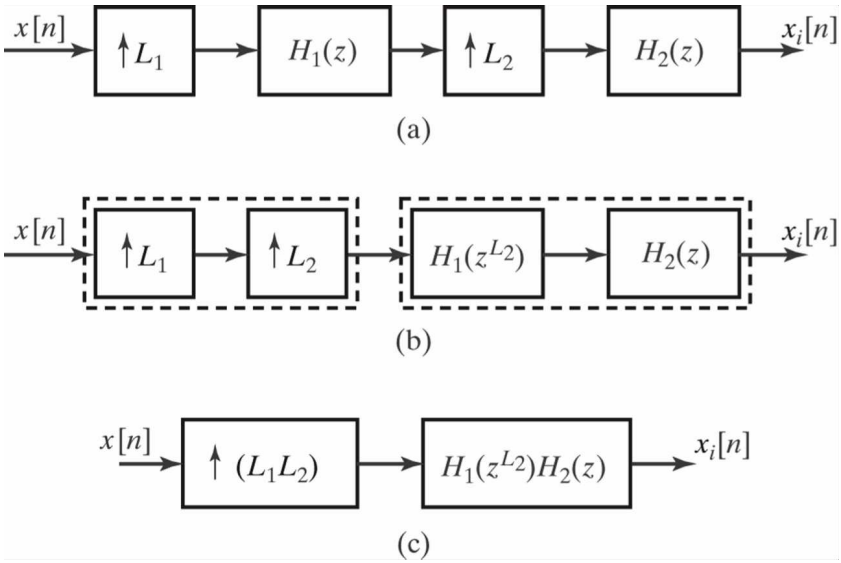
Approach (a): $x_a[n] = \{\underline{10}, 40, 70, 100, 80\}$ and
$y[n] = \{\underline{10}, 0, 40, 0, 70, 0, 100, 0, 80, 0\}$.

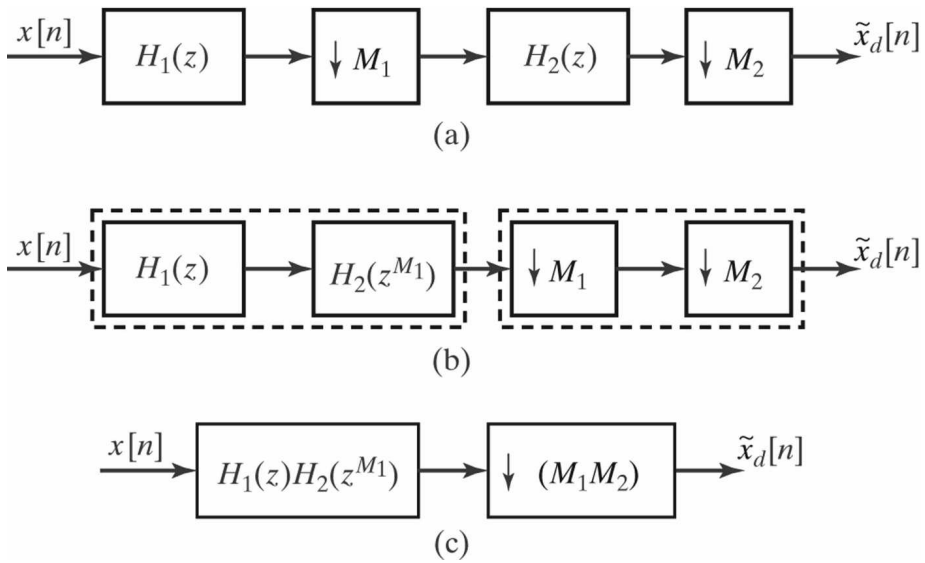
Approach (b): $x_b[n] = \{\underline{1}, 0, 2, 0, 3, 0, 4, 0\}$, $H(z^L) = 10 + 20z^{-2}$, and
$y[n] = \{\underline{10}, 0, 40, 0, 70, 0, 100, 0, 80, 0\}$.

Which approach is easier to implement?

Multistage Interpolation and Decimation

Equivalent structures:

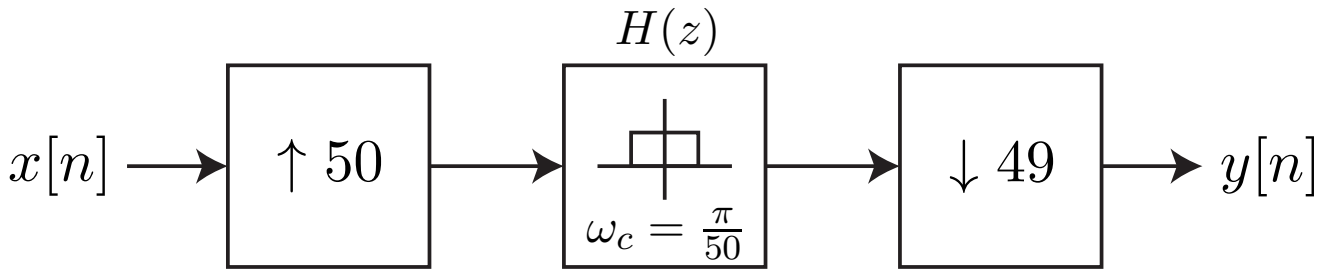




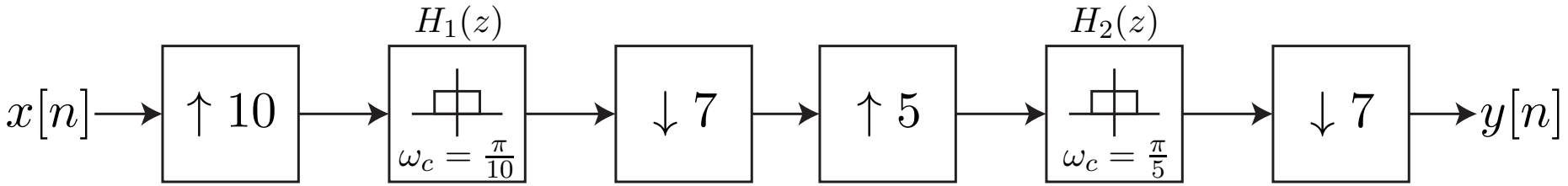
The main advantage of multistage interpolation and decimation is efficiency. Splitting interpolation and decimation into multiple steps avoids using computationally difficult narrowband filters. By up/downsampling in smaller steps, you can use simpler wideband lowpass filters in each stage.

Multistage Rational Sample Rate Conversion

Our earlier example to change the sample rate by $\frac{50}{49}$:



An equivalent (but more efficient) structure:



Try the Matlab function `upfirdn`.