Digital Signal Processing
Polyphase Implementation of Filtering

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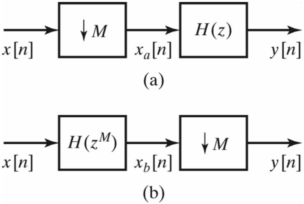
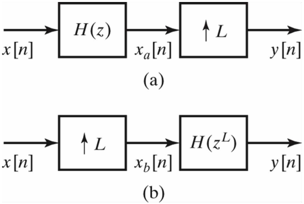
A Problem With Exchanging The Order of Up/Downsampling and Filtering

Recall our equivalent structures:





We prefer (a) in both cases because the filtering is done at the lower sampling rate.

This can be directly implemented in some cases, e.g. $M = 2$ and $H(z^M) = 1 + z^{-2}$. It is clear that $H(z) = 1 + z^{-1}$.

But what if $M = 2$ and $H(z^M) = 1 + z^{-1} + z^{-2}$? Does $H(z) = 1 + z^{-1/2} + z^{-1}$ make sense?

Polyphase Sequence Decompositions

Given an integer $M \geq 1$, we can decompose any discrete-time sequence $h[n]$ into M subsequences defined as

$$e_k[n] = h[nM + k]$$

for $k = 0, \dots, M-1$.

For example, suppose $h[n] = \{\underline{1}, 2, 3, 4, 5, 6\}$. An $M = 2$ polyphase decomposition results in

$$\begin{aligned} e_0[k] &= \{\underline{1}, 3, 5\} \\ e_1[k] &= \{\underline{2}, 4, 6\} \end{aligned}$$

An $M = 3$ polyphase decomposition results in

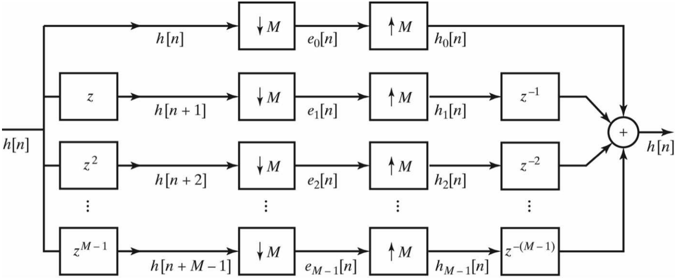
$$\begin{aligned} e_0[k] &= \{\underline{1}, 4\} \\ e_1[k] &= \{\underline{2}, 5\} \\ e_2[k] &= \{\underline{3}, 6\} \end{aligned}$$

Polyphase Components $\longrightarrow$ Original Sequence

To recover the original sequence from the polyphase components, we can

1. Upsample each polyphase component by M
2. Delay the k^{th} upsampled component by k samples.
3. Sum.

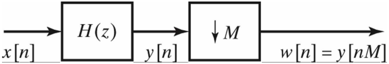
Decomposition/reconstruction:



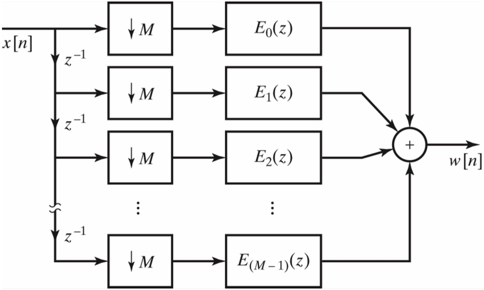
$$H(z) = \sum_{k=0}^{M-1} E_k(z^M) z^{-k} \longleftarrow \text{Each subfilter is a function of } z^M$$

Polyphase Decimation System

Suppose we had an N-coefficient FIR filtering system like



Note that $M-1$ of the M filter outputs are discarded. Is there a better way to do this?



Direct implementation requires $\approx N$ MACs per input sample.

Polyphase implementation:

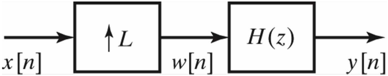
- Samples arrive at each polyphase filter at a rate of $\frac{1}{M}$ the original sampling rate.
- Each subfilter has $\frac{N}{M}$ coefficients.

Hence each subfilter requires $\approx \frac{1}{M} \cdot \frac{N}{M}$ MACs per input sample. The total is then $\approx \frac{N}{M}$ MACs per input sample.

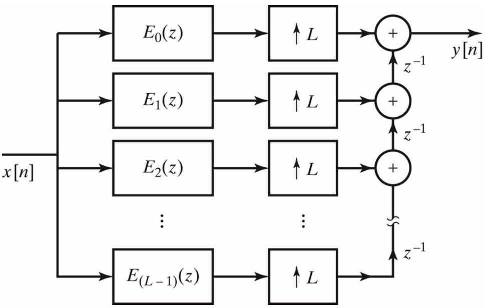
Computational savings achieved by filtering at the lower sampling rate.

Polyphase Interpolation System

Along the same lines, Suppose we had an N-coefficient FIR filtering system like



Note that $L-1$ of the L filter inputs are zero. Is there a better way to do this?



Direct implementation requires $\approx LN$ MACs per input sample.

Polyphase implementation:

- Samples arrive at each polyphase filter at the original sampling rate.
- Each subfilter has $\frac{N}{L}$ coefficients.

Hence each subfilter requires $\approx \frac{N}{L}$ MACs per input sample. The total is then $\approx N$ MACs per input sample.

Computational savings achieved by filtering at the lower sampling rate.

Remarks

- Exchanging the order of filtering and up/down-sampling can lead to equivalent systems with less computational requirements.
- Polyphase implementation allows this exchange to be possible for general filters.
- Matlab function `upfirdn` uses a polyphase interpolation structure.
- Also see Matlab function `resample`.

```
Y = resample(X,P,Q) resamples the sequence in vector X at
P/Q times the original sample rate using a polyphase
implementation.
```