# Digital Signal Processing
# Oversampled Analog to Digital Conversion with Direct Quantization

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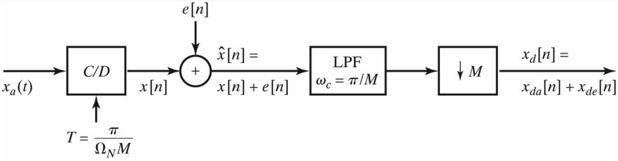
# Oversampled ADC System Model




Assumptions:

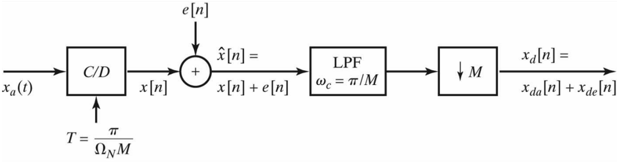
- The input signal is band limited so that $X_a(j\Omega) = 0$ for all $|\Omega| \geq \Omega_N$.
- The input signal is stationary with autocorrelation $\phi_{x_a x_a}(\tau)$ and power spectrum $\Phi_{x_a x_a}(j\Omega)$.
- The C/D block oversampled by a factor of $M$, i.e., $\Omega_s = 2M\Omega_N$.

We have seen previously that, when $M = 1$, the SNR (in dB) of a $B+1$ bit uniform quantizer with step size $\Delta = \frac{X_m}{2^B}$ is

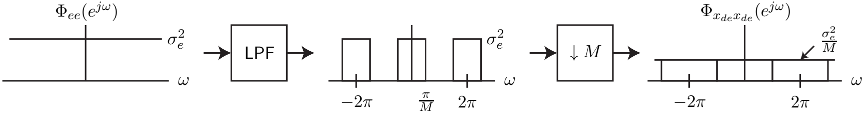
$$\mathrm{SNR}_Q = 10\log_{10}\left(\frac{\sigma_x^2}{\sigma_e^2}\right) = 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right)$$

Question: What happens when $M > 1$?

# Quantization Error Noise Analysis



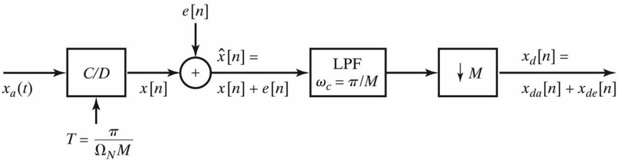

Under the usual white-noise statistical assumptions about the quantization error, we have $\Phi_{ee}(e^{j\omega}) = \sigma_e^2$. Hence the quantization noise propagates as



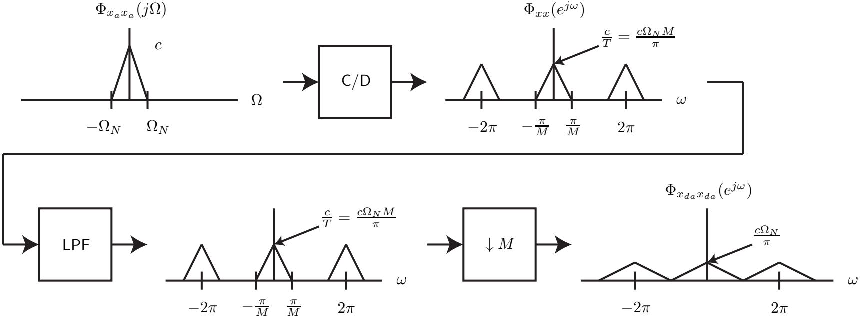

and the power of the quantization noise at the output of the downsampler can be computed as

$$\sigma_{x_{de}}^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\sigma_e^2}{M}\, d\omega = \frac{\sigma_e^2}{M}.$$

# Analysis of Signal Part (part 1 of 2)




We can analyze the signal component similarly:




Note that the signal power is unchanged through the LPF and downsampler.

# Analysis of Signal Part (part 2 of 2)

Note that the power of the CT input signal is

$$\sigma_{x_a}^2 = \frac{1}{2\pi} \int_{-\Omega_N}^{\Omega_N} \Phi_{x_a x_a}(j\Omega)\, d\Omega$$

The power of the sampled DT signal is

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{xx}(e^{j\omega})\, d\omega$$

but since

$$\Phi_{xx}(e^{jw}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \Phi_{x_a x_a}\left(j\frac{\omega}{T} - \frac{j2\pi k}{T}\right)$$

we have

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{T} \sum_{k=-\infty}^{\infty} \Phi_{x_a x_a}\left(j\frac{\omega}{T} - \frac{j2\pi k}{T}\right) d\omega = \frac{1}{2\pi} \int_{-\pi/M}^{\pi/M} \frac{1}{T} \Phi_{x_a x_a}\left(j\frac{\omega}{T}\right) d\omega.$$

We can substitute $\Omega = \frac{\omega}{T}$ and note that $d\omega = T d\Omega$ to write

$$\sigma_x^2 = \frac{1}{2\pi} \int_{-\Omega_N}^{\Omega_N} \Phi_{x_a x_a}\left(j\Omega\right)\, d\Omega = \sigma_{x_a}^2.$$

Hence the power of the analog input and the discrete-time signal are identical.

# Oversampled ADC With Direct Quantization SNR Analysis

We can compute the SNR (in dB) of a $B+1$ bit uniform quantizer with step size $\Delta = \frac{X_m}{2^B}$ and oversampling factor $M$ as

$$\begin{aligned}
\mathrm{SNR}_Q &= 10\log_{10}\left(\frac{\sigma^2_{x_{da}}}{\sigma^2_{x_{de}}}\right) \\
&= 10\log_{10}\left(\frac{\sigma^2_x}{\sigma^2_e/M}\right) \\
&= 10\log_{10}\left(\frac{12\cdot 2^{2B}\sigma^2_x M}{X_m^2}\right) \\
&= 6.02B + 10.8 - 20\log_{10}\left(\frac{X_m}{\sigma_x}\right) + 10\log_{10}(M)
\end{aligned}$$

Remarks:

- Main idea: Oversampling by $M$ doesn't affect the signal power but reduces the noise power by a factor of $M$.
- Doubling $M$ and keeping $B$ fixed gives a 3 dB SNR improvement.
- Quadrupling $M$ and using one less quantizer bit gives the same SNR.