Digital Signal Processing
Oversampled Digital to Analog Conversion with Noise Shaping

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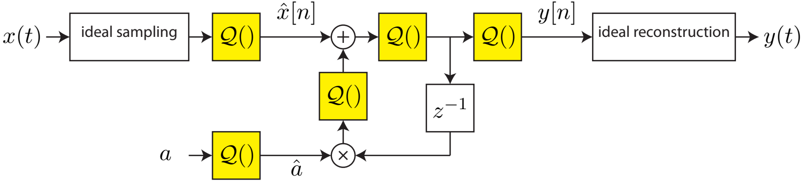
Digital to Analog Conversion Basics

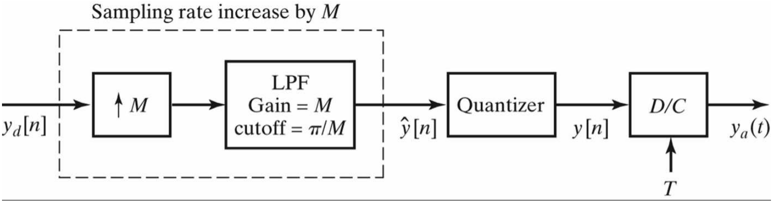
Often the precision of the signal processing algorithm is higher than the precision of the DAC, e.g., floating point processing and a 16-bit DAC.

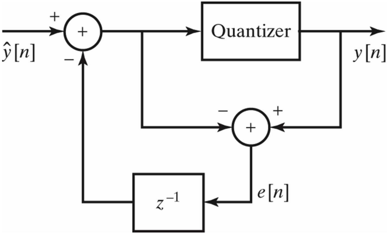


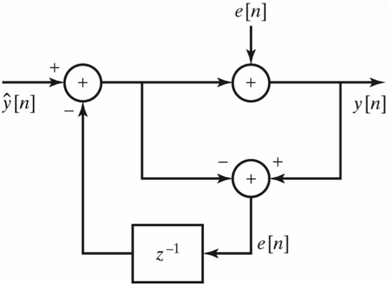
In these cases, we have to quantize the algorithm output before sending it to the DAC.

To minimize the effects of quantization noise (and to make the reconstruction filter easier to realize) we can oversample and shape the quantization noise just as we did with analog to digital conversion.

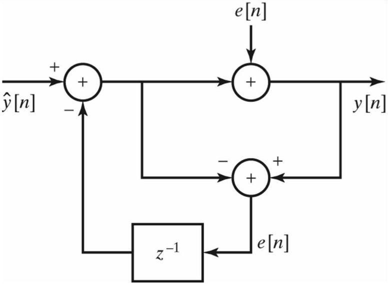
Oversamping and Noise Shaping







Analysis



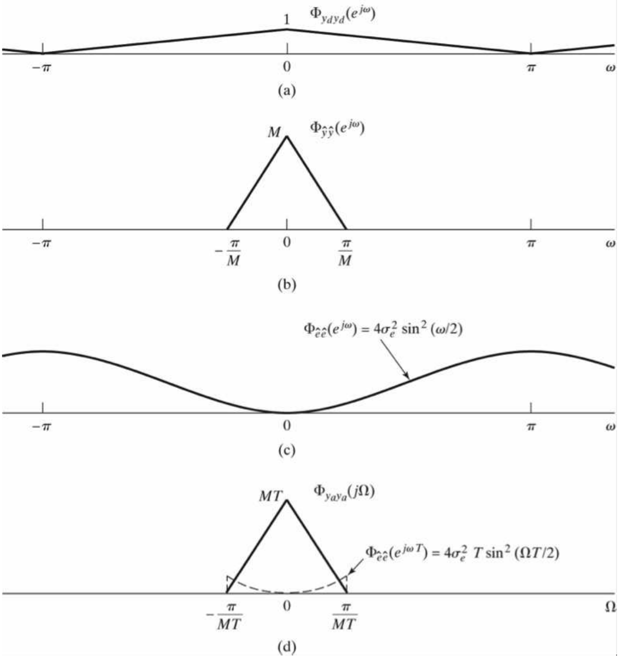
Note that

$$y[n] = \hat{y}[n] - e[n-1] + e[n]$$

We can easily derive the transfer function from $\hat{y}[n]$ to $y[n]$ as $H_y(z) = 1$ and we can also derive the transfer function from $\hat{e}[n]$ to $y[n]$ as

$$H_e(z) = 1 - z^{-1} \text{ with } |H(e^{j\omega})|^2 = 4\sin^2(\omega/2)$$

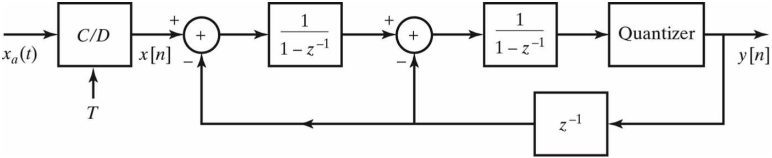
Hence, in this context, it should be clear that the noise shaping is identical to what we saw for analog-to-digital conversion.



1
$\Phi_{y_d y_d}(e^{j\omega})$
$-\pi$
0
π
ω
(a)
M
$\Phi_{\hat{y}\hat{y}}(e^{j\omega})$
$-\pi$
$-\frac{\pi}{M}$
0
$\frac{\pi}{M}$
π
ω
(b)
$\Phi_{\hat{e}\hat{e}}(e^{j\omega}) = 4\sigma_e^2 \sin^2(\omega/2)$
$-\pi$
0
π
ω
(c)
MT
$\Phi_{y_a y_a}(j\Omega)$
$\Phi_{\hat{e}\hat{e}}(e^{j\omega T}) = 4\sigma_e^2\, T \sin^2(\Omega T/2)$
$-\frac{\pi}{MT}$
0
$\frac{\pi}{MT}$
Ω
(d)

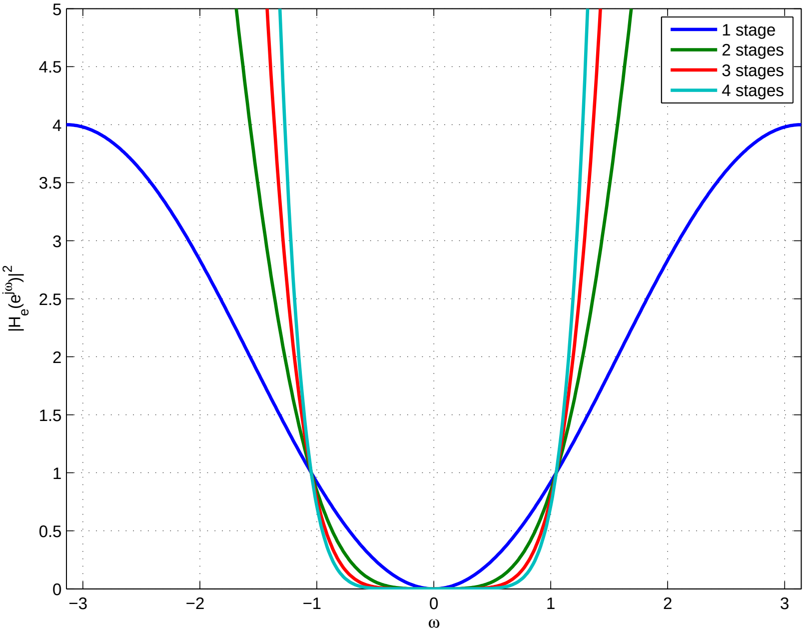
Multistage Noise Shaping

The ADC and DAC noise-shaping techniques we have looked at can also be applied with multiple stages. As an example, here is a block diagram of a two-stage ADC noise shaper:



You can verify for a p-stage system that the transfer function of the signal path remains unchanged and the transfer function of the quantization noise path becomes

$$H_e(z) = (1 - z^{-1})^p \text{ with } |H(e^{j\omega})|^2 = (2\sin(\omega/2))^{2p}.$$


1 stage
2 stages
3 stages
4 stages
$|H_e(e^{j\omega})|^2$
ω
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−3
−2
−1
0
1
2
3