Problem Setup

We are given a causal stable IIR low-pass filter

\[ H(z) = \frac{1-\alpha}{2} \frac{(1 + z^{-1})}{1 - \alpha z^{-1}} \]

with \( \alpha = 0.5 \). We can realize and analyze this filter in Matlab with

```matlab
w = pi*[0:0.001:1]; % normalized frequencies
alpha = 0.5; % LPF parameter
blp = (1-alpha)/2*[1 1]; % numerator coefficients
alp = [1 -alpha]; % denominator coefficients
hlp = freqz(blp,alp,w); % compute DTFT
glp = grpdelay(blp,alp,w); % compute group delay
```
Low-Pass Filter Frequency Response

![Graphs of magnitude response, phase response, and group delay of a low-pass filter.](image-url)
We will cascade a causal all-pass filter with the LPF to linearize the phase response and equalize the group delay in the passband. We can do this in Matlab with

```matlab
N = 4;  \% all-pass filter order
F = w(1:501)/pi;  \% normalized frequencies
edges = [0 1/2];  \% band-edge frequencies
Gd = max(glp)-glp(1:501);  \% desired group-delays of APF (>0)
[bap,aap] = iirgrpdelay(N,F,edges,Gd);  \% make all-pass filter
hap = freqz(bap,aap,w);  \% compute DTFT
gap = grpdelay(bap,aap,w);  \% compute group-delay
```
All-Pass Filter $z$-plane
All-Pass Filter Frequency Response

- **Magnitude Response**: The magnitude response is flat across the frequency range, indicating a constant gain for all frequencies.
- **Phase Response (rad)**: The phase response decreases linearly with frequency, indicating a linear phase characteristic.
- **Group Delay (samples)**: The group delay increases with frequency, indicating a delay that increases as the frequency increases.

The graphs show the frequency response characteristics of an all-pass filter, with the magnitude response being constant, and the phase response and group delay varying linearly with frequency.
Compute and Plot Cascaded Response

We can do this in Matlab with

```matlab
b = conv(blpr,bapr); % product of numerators
a = conv(alpr,aapr); % product of denominators
h = freqz(b,a,wr); % compute DTFT
G = grpdelay(b,a,wr); % compute group delay
subplot(3,1,1)
plot(w,abs(h));
ylabel('magnitude response');
subplot(3,1,2);
plot(w,unwrap(angle(h)));
ylabel('phase response (rad)');
subplot(3,1,3);
plot(w,G);
xlabel('normalized freq (rad/sample)');
ylabel('group delay (samples)');
```
Cascaded LPF-AP Frequency Response

- Frequency response:
  - Magnitude response
  - Phase response (rad)
- Group delay (samples)

Normalized freq (rad/sample): 0 to 3.5

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