Digital Signal Processing
Basic IIR Realization Structures

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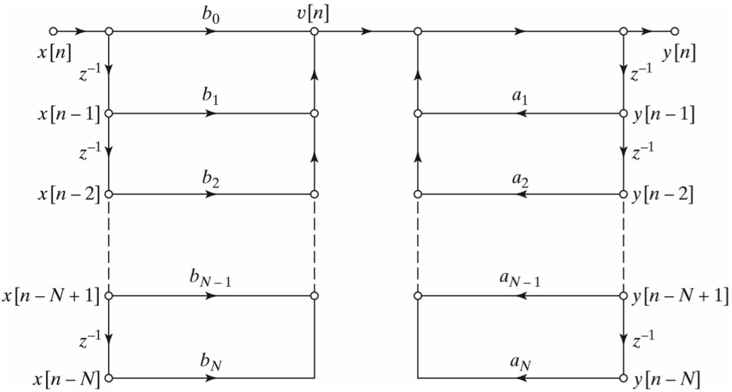
IIR Direct Form I

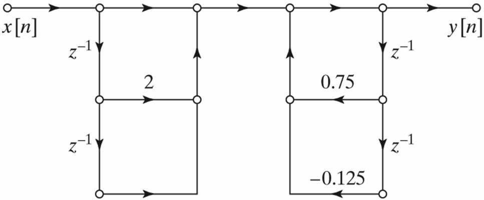
If $H(z) = \frac{P(z)}{Q(z)}$, we can implement this directly as a cascade of the feedforward part $P(z)$ and the feedback part $Q(z)$.



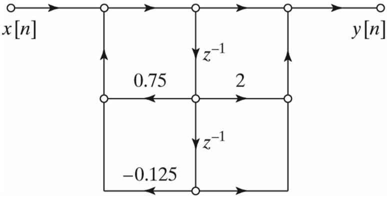
Changing the Order of the IIR DFI Sections

To get an equivalent structure, we can reverse the order of the sections (do feedback first, feedforward second). Example:

IIR DFI



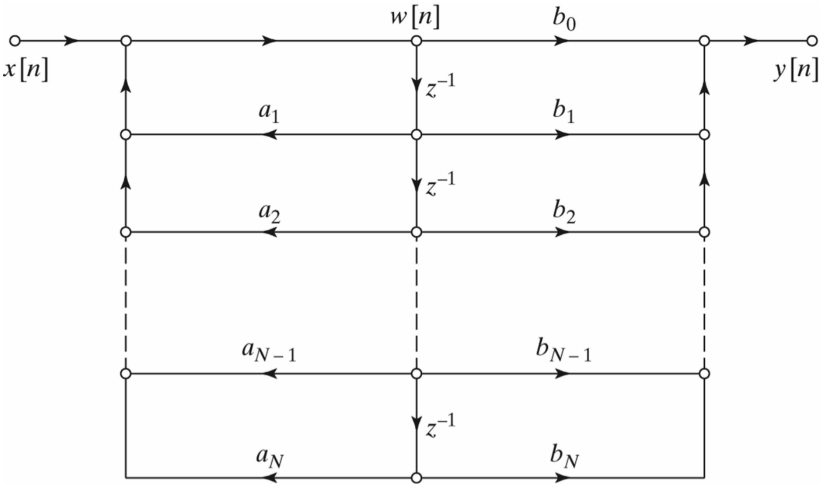
IIR-DFI swapped sections



Note the delays in the signal flow graph with swapped sections are no longer storing past inputs and past outputs.

IIR Direct Form II

By sharing the delays in our swapped sections structure, we arrive at a structure called IIR Direct Form II.



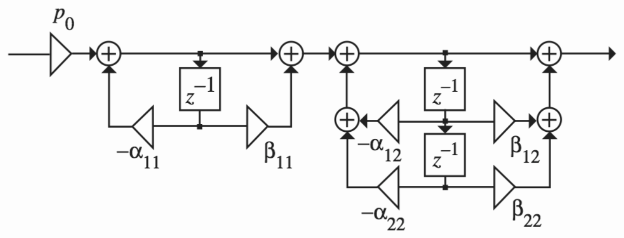
Pseudocode Implementation of IIR Direct Form II

```
<read in new input x[n]>

<shift intermediate values down, starting at bottom
  w[n-N+1] = w[n-N+2]
  w[n-N+2] = w[n-N+3]
  ...
>

<compute new intermediate value
  w[n] = x[n] - a[1]*w[n-1] - a[2]*w[n-2] - ...
>

<compute new output
  y[n] = b[0]*w[n] + b[1]*w[n-1] + b[2]*w[n-2] + ...
>
```

IIR Cascaded Direct Form II

Idea: factor transfer function and implement as a series/product of smaller cascaded transfer functions. Example:

$$H(z) = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}}{1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3}}$$
$$= \left(\frac{p_0(1 + \beta_{1,1} z^{-1})}{1 + \alpha_{1,1} z^{-1}} \right) \left(\frac{1 + \beta_{1,2} z^{-1} + \beta_{2,2} z^{-2}}{1 + \alpha_{1,2} z^{-1} + \alpha_{2,2} z^{-2}} \right)$$

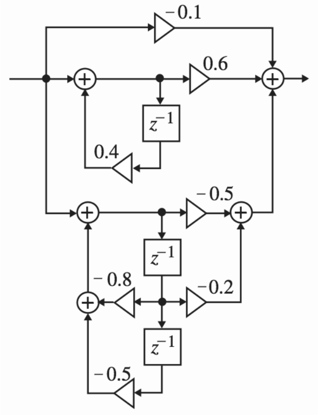


Both sections are implemented here as DF II. The ordering of the sections and the numerator/denominator pairings can affect performance when using finite precision. This is the default IIR structure in Matlab.

IIR Parallel Realization

One way to realize IIR filters in parallel is to do a partial fraction expansion. Example:

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{1-} + 0.18z^{-2} - 0.2z^{-3}} = -0.1 + \frac{0.6}{1 - 0.4z^{-1}} + \frac{-0.5 - 0.2z^{-1}}{1 + 0.8z^{-1} + 0.5z^{-2}}$$



Each subfilter is DFII. Can be useful for dividing computation across processors/cores.