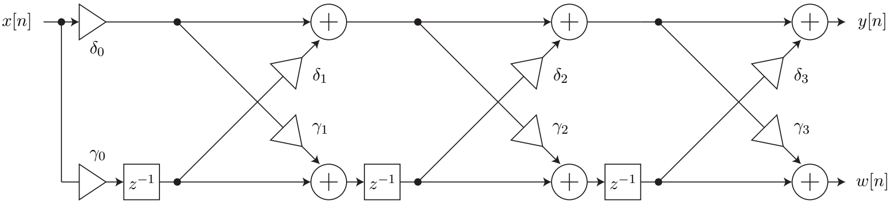
Digital Signal Processing
FIR Cascaded Lattice Filters

D. Richard Brown III

FIR Cascaded Lattice Structures



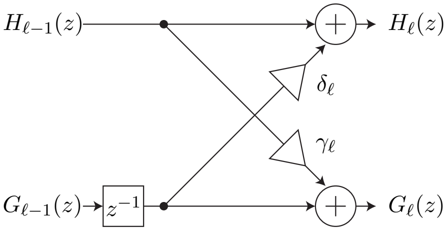
This approach can realize two FIR transfer functions:

$$H(z) = \frac{Y(z)}{X(z)} \text{ and } G(z) = \frac{W(z)}{X(z)}.$$

Given a lattice structure block diagram, how do we determine $H(z)$ and $G(z)$ (lattice $\rightarrow$ TF)?

Given a desired FIR $H(z)$ and $G(z)$ (or $h[n]$ and $g[n]$), how do we determine the lattice coefficients (TF $\rightarrow$ lattice)?

FIR Cascaded Lattice Structures (Lattice $\rightarrow$ TF)

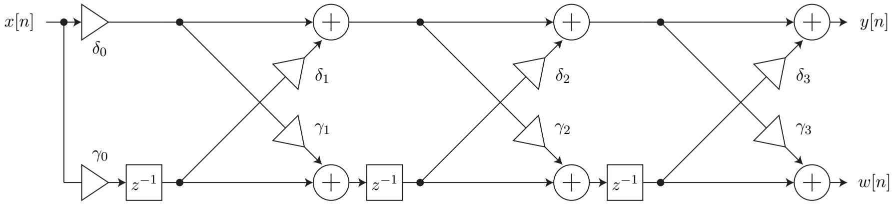


Analysis of single lattice segment:

$$H_\ell(z) = H_{\ell-1}(z) + z^{-1}\delta_\ell G_{\ell-1}(z)$$
$$G_\ell(z) = \gamma_\ell H_{\ell-1}(z) + z^{-1} G_{\ell-1}(z)$$

Since $H_0(z) = \delta_0 X(z)$ and $G_0(z) = \gamma_0 X(z)$, this recursion can be applied for $\ell = 1, 2, \dots, N$ to determine the transfer functions $H(z)$ and $G(z)$ from the lattice coefficients.

Lattice → TF Example



We can write

$$
\begin{aligned}
H_0(z) &= \delta_0 \\
G_0(z) &= \gamma_0 \\
H_1(z) &= H_0(z) + \delta_1 z^{-1} G_0(z) = \delta_0 + \delta_1 z^{-1} \gamma_0 \\
G_1(z) &= \gamma_1 H_0(z) + z^{-1} G_0(z) = \gamma_1 \delta_0 + z^{-1} \gamma_0 \\
H_2(z) &= H_1(z) + \delta_2 z^{-1} G_1(z) = \delta_0 + \delta_1 z^{-1} \gamma_0 + \delta_2 z^{-1} (\gamma_1 \delta_0 + z^{-1} \gamma_0) \\
G_2(z) &= \gamma_2 H_1(z) + z^{-1} G_1(z) = \gamma_2 (\delta_0 + \delta_1 z^{-1} \gamma_0) + z^{-1} (\gamma_1 \delta_0 + z^{-1} \gamma_0)
\end{aligned}
$$

and so on.

FIR Cascaded Lattice Structures (TF $\rightarrow$ Lattice)

To get a recursion for computing lattice coefficients given $H(z)$ and $G(z)$, we can redo our analysis, solving instead for $H_{\ell-1}(z)$ and $G_{\ell-1}(z)$, to get

$$\begin{aligned} H_{\ell-1}(z) &= K_\ell \left[H_\ell(z) - \delta_\ell G_\ell(z) \right] \\ G_{\ell-1}(z) &= K_\ell z \left[G_\ell(z) - \gamma_\ell H_\ell(z) \right] \end{aligned}$$

where $K_\ell = \frac{1}{1-\delta_\ell \gamma_\ell}$.

FIR Cascaded Lattice Structures (TF $\rightarrow$ Lattice)

To get a recursion for computing lattice coefficients given $H(z)$ and $G(z)$, we can redo our analysis, solving instead for $H_{\ell-1}(z)$ and $G_{\ell-1}(z)$, to get

$$H_{\ell-1}(z) = K_\ell \left[H_\ell(z) - \delta_\ell G_\ell(z)\right]$$
$$G_{\ell-1}(z) = K_\ell z \left[G_\ell(z) - \gamma_\ell H_\ell(z)\right]$$

where $K_\ell = \frac{1}{1-\delta_\ell\gamma_\ell}$.

Suppose, as an example, $\ell = 2$ with $H_\ell(z) = h_0 + h_1 z^{-1} + h_2 z^{-2}$ and $G_\ell(z) = g_0 + g_1 z^{-1} + g_2 z^{-2}$. Then

$$H_1(z) = K_2 \left[h_0 + h_1 z^{-1} + h_2 z^{-2} - \delta_2(g_0 + g_1 z^{-1} + g_2 z^{-2})\right]$$
$$G_1(z) = K_2 z \left[g_0 + g_1 z^{-1} + g_2 z^{-2} - \gamma_2(h_0 + h_1 z^{-1} + h_2 z^{-2})\right]$$

Note that $H_1(z)$ and $G_1(z)$ can only be order 1 transfer functions. Hence we need the z^{-2} and z^{+1} terms to vanish, which happens when

$$z^{-2} \text{ term in } H_1(z) \text{ vanishes} \quad \Leftrightarrow \quad h_2 - \delta_2 g_2 = 0 \quad \Leftrightarrow \quad \delta_2 = \frac{h_2}{g_2}$$
$$z^{+1} \text{ term in } G_1(z) \text{ vanishes} \quad \Leftrightarrow \quad g_0 - \gamma_2 h_0 = 0 \quad \Leftrightarrow \quad \gamma_2 = \frac{g_0}{h_0}$$

FIR Cascaded Lattice Structures (TF $\rightarrow$ Lattice)

Outline of procedure:

1. Set L equal to the filter order (impulse response length minus 1).
2. Set $H_L(z) = H(z)$ and $G_L(z) = G(z)$.
3. Set $\ell = L$.
4. Solve for δ_ℓ and γ_ℓ.
5. Compute $K_\ell = \frac{1}{1-\delta_\ell\gamma_\ell}$.
6. Compute $H_{\ell-1}(z)$ and $G_{\ell-1}(z)$. Both transfer functions should be order $\ell - 1$. Note coefficients are affected by K_ℓ (you can't just say $H_{\ell-1}(z)$ is equal to the first $\ell - 1$ coefficients of $H_\ell(z)$ or that $G_{\ell-1}(z)$ is equal to the first $\ell - 1$ coefficients of $G_\ell(z)$).
7. Decrement ℓ.
8. If $\ell \geq 1$ go to step 4.
9. The final gains at the input are simply $\delta_0 = H_0(z)$ and $\gamma_0 = G_0(z)$ (these are both zero-order transfer functions).

TF $\rightarrow$ Lattice Example

Suppose $H(z) = 1 + 2z^{-1} + 3z^{-2}$ and $G(z) = 4 + 5z^{-1} + 6z^{-2}$.

1. We set $\ell = L = 2$ and solve $\delta_2 = \frac{3}{6} = \frac{1}{2}$ and $\gamma_2 = \frac{4}{1} = 4$.
2. We compute $K_2 = \frac{1}{1-2} = -1$.
3. We compute

$$H_1(z) = -1 \cdot \left[1 + 2z^{-1} + 3z^{-2} - \frac{1}{2}(4 + 5z^{-1} + 6z^{-2})\right] = 1 + \frac{1}{2}z^{-1}$$

   and

$$G_1(z) = -1z\left[4 + 5z^{-1} + 6z^{-2} - 4(1 + 2z^{-1} + 3z^{-2})\right] = 3 + 6z^{-1}$$

4. Now set $\ell = 1$ and solve $\delta_1 = \frac{\frac{1}{2}}{6} = \frac{1}{12}$ and $\gamma_1 = \frac{3}{1} = 3$.
5. Compute $K_1 = \frac{1}{1-\frac{3}{12}} = \frac{4}{3}$.
6. Compute

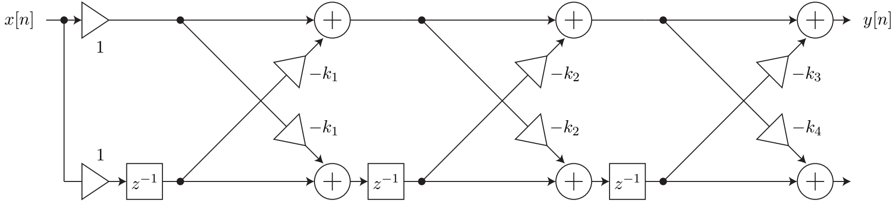
$$H_0(z) = \frac{4}{3} \cdot \left[1 + \frac{1}{2}z^{-1} - \frac{1}{12}(3 + 6z^{-1})\right] = 1 = \delta_0$$

   and

$$G_0(z) = \frac{4}{3}z\left[3 + 6z^{-1} - 3(1 + \frac{1}{2}z^{-1})\right] = 6 = \gamma_0$$

Special Case: Symmetric Lattice

Often, we only need a single transfer function and can use the symmetric lattice



The procedures for TF → lattice and lattice → TF are similar to what we've covered. Also see Matlab functions `tf2latc` and `latc2tf`.