Digital Signal Processing
Effect of Realization Structure on Second-Order Pole Locations

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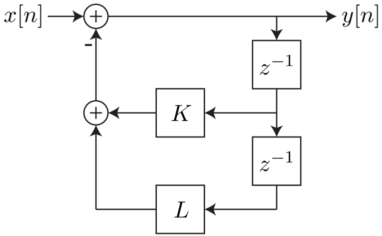
Direct Form II Second Order All-Pole System

Consider the following DF-II realization of an all-pole second-order IIR filter:

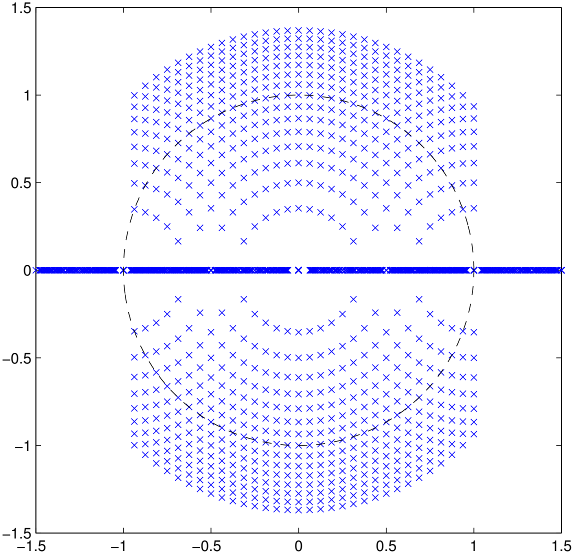


The transfer function is

$$H(z) = \frac{1}{1 + Kz^{-1} + Lz^{-2}}$$

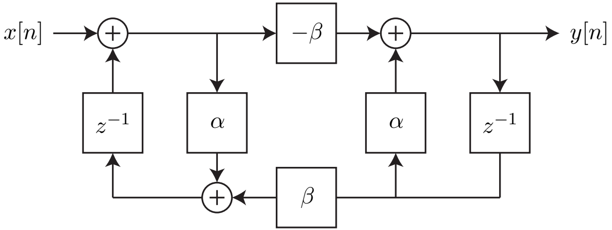
When we quantize the coefficients K and L, there are only a finite number of possible pole locations that we can achieve.

Second Order DF-II Pole Locations ($B+1=5$, $q_a=3$)



Second Order Coupled Form

Now consider the following **coupled-form** realization of an all-pole second-order IIR filter:

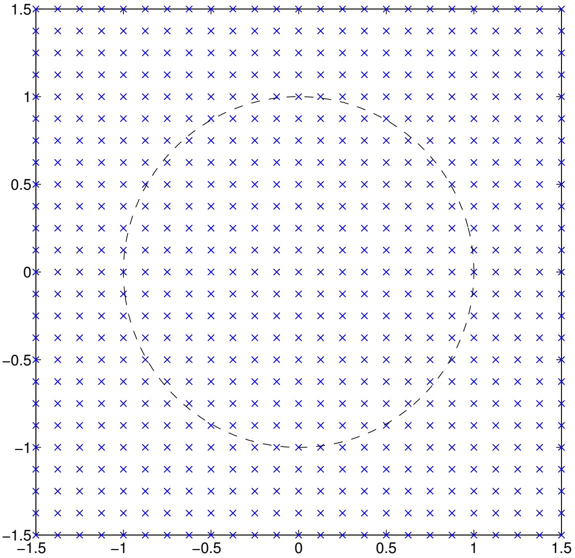


The transfer function is

$$H(z) = \frac{\beta}{1 - 2\alpha z^{-1} + (\alpha^2 + \beta^2) z^{-2}}$$

Again, when we quantize the coefficients α and β, there are only a finite number of possible pole locations that we can achieve.

Second Order Coupled-Form Pole Loc. ($B+1=5$, $q_a=3$)



Second Order Coupled-Form Pole Locations

Remarks:

1. Pole locations are uniformly distributed on z-plane, which may be preferred.
2. Suitable for any type of IIR filter, e.g., lowpass, highpass, ...
3. Quantized pole displacement is easily bounded.
4. Why not always use the coupled form? Is there any disadvantage?
5. This approach can also be used to determine quantized zero locations, although IIR transfer functions are usually less sensitive to numerator quantization.