ECE503 Spring 2014 Quiz 10

Your Name: ______________________________ ECE Box Number: _________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points total. Suppose you wish to convert the continuous-time filter

$$H_c(s) = \frac{1}{s+1}$$

to a discrete-time filter $H(z)$. For the following questions, assume a sampling period $T_d = 0.1$ and simplify your answers as much as possible.

   (a) 15 points. Compute $H(z)$ using the impulse invariance method so that $h[n] = T_d h_c(nT_d)$.

   (b) 15 points. Compute $H(z)$ using the bilinear transform method.

2. 40 points. Given the following specifications for a discrete-time low pass filter

$$\frac{1}{\sqrt{2}} \leq |H(e^{j\omega})| \leq 1 \text{ for } 0 \leq \omega \leq \pi/3 \quad \text{(passband)}$$
$$|H(e^{j\omega})| \leq 0.25 \text{ for } 2\pi/3 \leq \omega \leq \pi \quad \text{(stopband)}$$

determine the minimum order N and cutoff frequency Ω_c of a continuous-time Butterworth lowpass filter satisfying the specifications assuming the bilinear transform will be used to convert the continuous-time lowpass filter to discrete time. Assume a sampling period $T_d = 1$. Design your filter to match the stop band specification exactly. You do not need to calculate $H_c(s)$ or $H(z)$; just specify the minimum order N and cutoff frequency Ω_c and show your reasoning.

3. 30 points. Observe that the system $H_c(s)$ in problem 1 is a first-order Butterworth filter with $\Omega_c = 1$. Suppose this lowpass filter was designed from a specification with passband frequency edge $\Omega_p = 0.5$ and stop band frequency edge $\Omega_s = 2$. Transform $H_c(s)$ to a highpass filter $H_{\text{hp}}(s)$ with passband frequency edge $\hat{\Omega}_p = 20$. Also calculate the resulting stop band frequency edge $\hat{\Omega}_s$. If you were to then convert $H_{\text{hp}}(s)$ to a discrete-time filter, which method(s) would be appropriate (and why): impulse invariance or bilinear transform?

1. a) We can use eq. 7.10 to write $(A_K = 1,\ s_K = -1,\ N=1)$

$$H(z) = \frac{T_d}{1 - e^{-T_d} z^{-1}} = \frac{0.1}{1 - e^{-0.1} z^{-1}}$$

b) $H(z) = \dfrac{1}{\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1}$ via bilinear transform

$$= \frac{\frac{1}{20}}{\frac{1-z^{-1}}{1+z^{-1}} + \frac{1}{20}} = \frac{\frac{1}{20}(1+z^{-1})}{\frac{21}{20} - \frac{19}{20} z^{-1}} = \frac{1+z^{-1}}{21 - 19 z^{-1}}$$

$$= \frac{\frac{1}{21}(1+z^{-1})}{1 - \frac{19}{21} z^{-1}}$$

2. step 1: pre-warp frequencies : $\Omega = 2\tan\left(\frac{\omega}{2}\right)$

$$\Rightarrow \Omega_p = 2\tan(\pi/6) = 1.1547$$

$$\Omega_s = 2\tan(\pi/3) = 3.4641$$

passband : $\dfrac{1}{1 + \left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} \ge \dfrac{1}{2} \Rightarrow \left(\dfrac{\Omega_p}{\Omega_c}\right)^{2N} \le 1$

Stopband : $\dfrac{1}{1 + \left(\frac{\Omega_s}{\Omega_c}\right)^{2N}} \le \dfrac{1}{16} \Rightarrow \left(\dfrac{\Omega_s}{\Omega_c}\right)^{2N} \ge 15$

$$2N\left[\log \Omega_p - \log \Omega_c\right] = 0$$

$$2N\left[\log \Omega_s - \log \Omega_c\right] = \log 15$$

$$2N\left[\log \Omega_p - \log \Omega_s\right] = -\log 15 \quad \Rightarrow N = 1.23;\ \text{set } N = 2$$

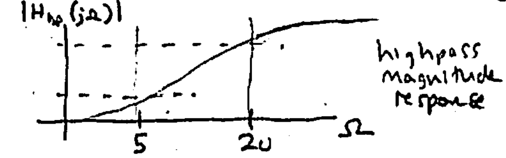
match stopband exactly : $\left(\dfrac{\Omega_s}{\Omega_c}\right)^4 = 15 \Rightarrow \dfrac{\Omega_s}{\Omega_c} = \sqrt[4]{15} \Rightarrow \Omega_c = 1.7602$

3. Lowpass $\rightarrow$ highpass : $s \rightarrow \Omega_p \hat{\Omega}_p \cdot \frac{1}{s}$ $\qquad \Omega_p = 0.5,\ \hat{\Omega}_p = 20$

We have $\Omega_p \hat{\Omega}_p = 10$, so $s \rightarrow \frac{10}{s}$

$$H_{hp}(s) = H_c\left(\frac{10}{s}\right) = \frac{1}{\frac{10}{s} + 1} = \frac{s}{10 + s}$$

The new stop band frequency $\hat{\Omega}_s = \dfrac{\Omega_p \hat{\Omega}_p}{\Omega_s} = \dfrac{10}{2} = 5$



Note that only the bilinear transform is appropriate here since this highpass filter is not bandlimited.

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