ECE503 Spring 2014 Quiz 11

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 40 points. Suppose $x[n]$ is a real-valued length-N sequence defined for $n = 0, 1, \ldots, N-1$ with N even and
$$y[n] = (-1)^n x[n]$$
is another length-N sequence. Explicitly relate $Y[k] = \mathsf{DFT}_N\{y[n]\}$ and $X[k] = \mathsf{DFT}_N\{x[n]\}$. Hint: It may be useful to observe that $(-1)^n = e^{-j\pi n} = W_2^n$.

2. 30 points. Suppose
$$x[n] = \begin{cases} n+1 & n = 0, \ldots, 9 \\ 0 & \text{otherwise} \end{cases}$$
and let $Y[k] = X(e^{j\omega})|_{\omega=2\pi k/8}$ for $k = 0, \ldots, 7$. Compute $y[n]$ for $n = 0, \ldots, 7$.

3. 30 points. Suppose you have length-4 sequences $x[n] = \{\underline{1}, 0, 2, 1\}$ and $y[n] = \{\underline{a}, 0, b, 0\}$ with a and b both unknown. The 4-point circular convolution of these sequences results in $z[n] = \{\underline{1}, -1, -1, 1\}$. Determine valid values for a and b. Is your answer unique?

1. can use property 8 on table 8.2

$$x_1[n] = (-1)^n \longleftrightarrow X_1[k] = \{0, \cdots, 0, \underset{\uparrow}{N}, 0, \ldots 0\}$$

at index $k = \frac{N}{2}$

$$x_2[n] = x[n] \longleftrightarrow X_2[k] = X[k]$$

$$x_1[n]\, x_2[n] \longleftrightarrow \frac{1}{N} \sum_{\ell=0}^{N-1} X_1[\ell]\, X_2[((k-\ell))_N]$$

(circular convolution of $X_1[k]$ and $X_2[k]$)

This circular convolution is just a circular shift of $X[k]$ by $\frac{N}{2}$

Note that the N and $\frac{1}{N}$ cancel. So

$$\boxed{Y[k] = X[((k - \tfrac{N}{2}))_N]}$$

You can also get this from property 6 by noting that

$$W_N^{-\frac{N}{2}n} = e^{j\pi n} = (-1)^n$$

2. Since the DFT length $M=8$ is less than $N=10$, sampling $X(e^{j\omega})$ will result in time-domain aliasing.

We have $y[n] = \sum_{r=-\infty}^{\infty} x[n+8r]$

$$y[0] = \cdots x[-8] + x[0] + x[8] + x[16] + \cdots = 1 + 9 = 10$$

$$y[1] = \cdots x[-7] + x[1] + x[9] + x[17] + \cdots = 2 + 10 = 12$$

$$y[2] = \cdots x[-6] + x[2] + x[10] + x[18] + \cdots = 3$$

$\vdots$

$$y[7] = \cdots x[-1] + x[7] + x[15] + x[23] + \cdots = 8$$

Hence $y[n] = \{\underline{10}, 12, 3, 4, 5, 6, 7, 8\}$ $n = 0, \ldots, 7$

3. Set up circular conv. matrix

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ a \\ 0 \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

first row implies $a = 1$

third row implies $b = -1$

Rows 2 and 4 consistent.

Answer is unique.