ECE503 Spring 2014 Quiz 12

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points total. Suppose you have a signal $x[n]$ which is known to be zero except on $0 \leq n \leq 9$ and another signal $h[n]$ which is known to be zero except on $0 \leq n \leq 4$. Let

$$y[n] = x[n] * h[n]$$

where $*$ is the usual linear convolution and let

$$\hat{y}[n] = x[n] \textcircled{\scriptsize N} h[n]$$

where $\textcircled{\scriptsize N}$ is an N-point circular convolution.

   (a) 10 points. What is the minimum value of N such that $\hat{y}[n] = y[n]$ for all n?

   (b) 20 points. Suppose we don't need the circular convolution to match the linear convolution for all n but instead only require $\hat{y}[n] = y[n]$ for $n = 5, 6, 7$. What is the minimum value of N such that $\hat{y}[n] = y[n]$ for $n = 5, 6, 7$?

2. 40 points total. Suppose you sample a continuous-time signal $x(t)$ a sampling frequency of $f_s = 32$ kHz and you know that no aliasing occurred. You apply a rectangular window of length $N = 128$ and compute the 128-point DFT of the resulting finite-length signal $\{x[0], \ldots, x[127]\}$.

   (a) 20 points. Strong peaks in the DFT magnitude are observed at $k = 10$ and $k = 118$. What are the continuous-time frequencies associated with these peaks? Your answer should be in Hertz.

   (b) 20 points. Suppose $x(t) = \cos(2\pi \cdot 4000t) + 0.01\cos(2\pi \cdot 4250t)$. Will the weaker tone at 4250 Hz be visible in the DFT? Explain why or why not.

3. 30 points. Suppose need to design a length-256 Kaiser window with relative sidelobe amplitude at least 40 dB below the main lobe. What is the narrowest mainlobe you can achieve while meeting this sidelobe specification? How does this compare to the mainlobe width and relative sidelobe amplitude (in dB) of a rectangular window with the same length?

1. a) $x[n]$ is length 10
$h[n]$ is length 5

$x[n] * h[n]$ will be length 14

$\Rightarrow \boxed{N \geq 14}$ for the circular conv to match the linear conv.

b) In this case, we can have some time-domain aliasing

N=13 causes one sample of aliasing ($\hat{y}[0] \neq y[0]$)
N=12 causes two samples of aliasing ($\hat{y}[1] \neq y[1], \ldots$)
⋮
N=9 causes five samples of aliasing ($\hat{y}[4] \neq y[4], \ldots$)

$\Rightarrow \boxed{N \geq 9}$ for $\hat{y}[n] = y[n]$ $n = 5, 6, 7$

2. $\omega = \frac{2\pi K}{N}$; $\Omega = \frac{\omega}{T}$; $f = \frac{\Omega}{2\pi}$ $\Rightarrow$ $f = \frac{K}{NT} = \frac{K}{N} f_s$

a) when $K=10$ we have $\boxed{f = \frac{10}{128} \cdot 32000 = 2500\,Hz}$

for $K=118$, since we know there is no aliasing, this must correspond to a negative frequency

$$\boxed{f = \frac{118-128}{128} \cdot 32000 = -2500\,Hz}$$

Hence it appears we have sampled a sinusoidal signal at $f = 2500\,Hz$ (although, since the resolution of the DFT is 250 Hz, this may not be the exact frequency of the signal; there may also be multiple closely spaced frequencies present)

b) $\boxed{\text{Yes.}}$ Both tones are exactly on DFT indices ($K=16$ and $k=17$) and since we are using a rectangular window, they do not interfere with each other.

3. From (10.14) we have

$$L \approx \frac{24\pi(A_{SL}+12)}{155\Delta_{ML}} + 1$$

we have $L = 256$
$A_{SL} = 40$

Solve for $\Delta_{ML} = \frac{24\pi(A_{SL}+12)}{155(L-1)} = 0.0992$

For a rectangular window, the mainlobe width is

$\Delta_{ML} = \frac{4\pi}{L} = 0.0491$, which is about half as wide as the Kaiser window

The relative sidelobe amplitude of the rectangular window is, however, only 13.26 dB below the main lobe, which is much worse.