ECE503 Spring 2014 Quiz 2

Your Name: ______________________________ ECE Box Number: _________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points. Compute the z-transform and the region of convergence (ROC) of the sequence defined as
$$x[n] = \begin{cases} 1 & n = 0, 2, 4, \ldots \\ 0 & \text{otherwise.} \end{cases}$$
Simplify your answer as much as possible to receive full credit.

2. 50 points total. Given
$$X(z) = \frac{3 - z^{-1}}{1 - 3z^{-1} + 2z^{-2}}.$$
   (a) 10 points. List *all* of the poles and zeros of $X(z)$.
   (b) 20 points. Determine all possible regions of convergence for $X(z)$.
   (c) 20 points. Compute the sequence $x[n]$ corresponding to each region of convergence.

3. 20 points. Recall that the z-transform of the unit-step function $x[n] = u[n]$ is given as
$$X(z) = \frac{1}{1 - z^{-1}} \qquad |z| > 1$$
and the DTFT of the unit step function is given as
$$X(e^{j\omega}) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) + \frac{1}{1 - e^{-j\omega}}.$$
Note that $X(e^{j\omega}) \neq X(z)|_{z=e^{j\omega}}$ in this case. Explain why.

1. $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0,2,4,\ldots}^{\infty} z^{-n} = \sum_{m=0,1,2,\ldots}^{\infty} z^{-2m} = \sum_{m=0}^{\infty} (z^2)^{-m}$

$$(1-z^{-2}) \sum_{m=0}^{N} (z^2)^{-m} = (1-z^{-2}) \left[1 + z^{-2} + z^{-4} + \ldots + z^{-2N} \right]$$

$$= 1 - z^{-2N-2}$$

so $\displaystyle\sum_{m=0}^{N} (z^2)^{-m} = \frac{1 - z^{-2N-2}}{1 - z^{-2}}$

and $\displaystyle\sum_{m=0}^{\infty} (z^2)^{-m} = \frac{1}{1 - z^{-2}}$ if $|z| > 1$

Hence $\boxed{X(z) = \frac{1}{1-z^{-2}} \quad \text{with ROC } |z| > 1}$

2. a) $X(z) = \dfrac{3 - z^{-1}}{(1-2z^{-1})(1-z^{-1})}$

$$\boxed{\begin{array}{l} \text{poles at } z = 1 \text{ and } z = 2 \\ \text{zeros at } z = \tfrac{1}{3} \text{ and } \underline{z = 0} \end{array}}$$

To see the pole at $z = 0$, you can rewrite $X(z)$ as $X(z) = \dfrac{3z^2 - z}{(z-2)(z-1)}$

clearly $X(z) = 0$ when $z = 0$

b) $$\boxed{\begin{array}{ll} ROC_A : & |z| < 1 \\ ROC_B : & 1 < |z| < 2 \\ ROC_C : & |z| > 2 \end{array}}$$

each ROC is bounded by the poles

$ROC_A \rightarrow$ left-sided sequence

$ROC_B \rightarrow$ two-sided "

$ROC_C \rightarrow$ right sided "

c) partial fraction expansion

$$X(z) = \frac{A_1}{1-2z^{-1}} + \frac{A_2}{1-z^{-1}}$$

$$A_1 = (1-2z^{-1})X(z)\Big|_{z=2} = \frac{3-z^{-1}}{1-z^{-1}}\Big|_{z=2} = \frac{3-\frac{1}{2}}{1-\frac{1}{2}} = 5$$

$$A_2 = (1-z^{-1})X(z)\Big|_{z=1} = \frac{3-z^{-1}}{1-2z^{-1}}\Big|_{z=1} = \frac{3-1}{1-2} = -2$$

$$\boxed{\begin{array}{lll} ROC_A: |z|<1 & \Rightarrow & x[n] = -5\cdot 2^n u[-n-1] + 2\cdot u[-n-1] \\ ROC_B: 1<|z|<2 & \Rightarrow & x[n] = -5\cdot 2^n u[-n-1] - 2u[n] \\ ROC_c: |z|>2 & \Rightarrow & x[n] = 5\cdot 2^n u[n] - 2u[n] \end{array}}$$

3. $X(e^{jw}) = X(z)\big|_{z=e^{jw}}$ only when the ROC of $X(z)$ includes the unit circle. That is not the case here, since the ROC is $|z|>1$.

In general, $X(z)$ is an analytic function of z and hence can't have discontinuous functions like $\delta(z)$.