ECE503 Spring 2014 Quiz 3

Your Name: ____________________________  ECE Box Number: ______

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 70 points total. Suppose you have a discrete-time system defined by the difference equation

\[ y[n] = x[n] - x[n - 1] - y[n - 1] \]

(a) 20 points. Compute the impulse response of this system.

(b) 30 points. Compute the response of this system to the input \( x[n] = nu[n] \) assuming relaxed initial conditions.

(c) 20 points. Repeat part (b) with the initial condition \( y[-1] = 1 \).

2. 30 points. Consider the ideal sampling and reconstruction system shown in Fig. 1. Suppose the input to the system is \( x_c(t) = \cos(2\pi \cdot 1000t) \) and the sampling rate is \( f_s = \frac{1}{T} = 10 \) kHz. Further suppose that, instead of the usual lowpass reconstruction filter, we use a reconstruction filter \( h_r(t) \) with ideal bandpass frequency response

\[
H_r(j\Omega) = \begin{cases} 
T & 2\pi \cdot 5000 \leq |\Omega| \leq 2\pi \cdot 10000 \\
0 & \text{otherwise.}
\end{cases}
\]

Determine the output \( x_r(t) \).

![Diagram](image)

Figure 1: Ideal sampling and reconstruction system.
\[ a) \quad y[n] + y[n-1] = x[n] - x[n-1] \]

\[(1 + z^{-1}) Y(z) = (1 - z^{-1}) X(z)\]

\[H(z) = \frac{1 - z^{-1}}{1 + z^{-1}}\]

\[h[n] = (-1)^n u[n] - (-1)^{n-1} u[n-1]\]

\[= (-1) (-1)^{n-1} u[n] - (-1)^{n-1} u[n-1]\]

\[= \delta[n] - (-1)^{n-1} u[n-1] - (-1)^{n-1} u[n-1]\]

\[= \delta[n] - 2(-1)^{n-1} u[n-1]\]

\[b) \quad x[n] = nu[n] \quad \Rightarrow \quad X(z) = \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > 1\]

\[Y(z) = H(z) X(z) = \frac{z^{-1}}{(1 + z^{-1})(1 - z^{-1})} \quad |z| > 1\]

\[= \frac{A_1}{1 + z^{-1}} + \frac{A_2}{1 - z^{-1}} \quad A_1 + A_2 = 0\]

\[-A_1 + A_2 = 1\]

\[Y(z) = -\frac{\sqrt{2}}{1 + z^{-1}} + \frac{\sqrt{2}}{1 - z^{-1}} \quad |z| > 1\]

\[y[n] = -\frac{1}{2} (-1)^n u[n] + \frac{1}{2} u[n]\]

\[= \frac{1}{2} \left[ (-1)^{n+1} + 1 \right] u[n] = \begin{cases} \frac{3}{2}, & n \in \mathbb{Z} \\ 0, & n \leq 0 \end{cases}\]

\[c) \quad \text{With non-zero initial conditions and } x[n] = 0 \quad \forall n \leq 0\]

we have

\[Y(z) = \frac{-y[-1]}{1 + z^{-1}} + H(z) X(z) \quad |z| > 1\]

\(\frac{\text{Zero input response}}{\text{We already computed this}}\)

\(\text{(Zero initial conditions response) continued...}\)
hence
\[ y[n] = (-1)^n (-1)^n u[n] + \frac{1}{2} ( (-1)^{n+1} + 1 ) u[n] \]
\[ = ( (-1)^n \frac{1}{2} (-1)^{n+1} + \frac{1}{2} ) u[n] \]
\[ = \left\{ 1, -1, 2, -1, 2, \ldots \right\} \]
\[ y[1] \neq y[0] \]

2. Sketch spectra...

\[ \chi_s(j\Omega) \]

\[ \Omega_s = 2\pi \cdot 10^3 \text{ rad/sec} \]

\[ H_r(j\Omega) \]

\[ \chi_r(j\Omega) \]

\text{all other frequencies blocked}

Hence \[ \chi_r(t) = \cos(2\pi \cdot 9000 t) \] By recovering with a bandpass filter, we have shifted the input from 1kHz to 9kHz.