ECE503 Spring 2014 Quiz 3

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 70 points total. Suppose you have a discrete-time system defined by the difference equation

$$y[n] = x[n] - x[n-1] - y[n-1]$$

   (a) 20 points. Compute the impulse response of this system.

   (b) 30 points. Compute the response of this system to the input $x[n] = nu[n]$ assuming relaxed initial conditions.

   (c) 20 points. Repeat part (b) with the initial condition $y[-1] = 1$.

2. 30 points. Consider the ideal sampling and reconstruction system shown in Fig. 1. Suppose the input to the system is $x_c(t) = \cos(2\pi \cdot 1000t)$ and the sampling rate is $f_s = \frac{1}{T} = 10$ kHz. Further suppose that, instead of the usual lowpass reconstruction filter, we use a reconstruction filter $h_r(t)$ with ideal *bandpass* frequency response

$$H_r(j\Omega) = \begin{cases} T & 2\pi \cdot 5000 \leq |\Omega| \leq 2\pi \cdot 10000 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the output $x_r(t)$.

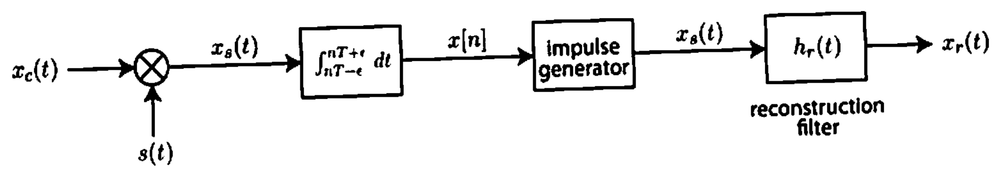


Figure 1: Ideal sampling and reconstruction system.

1. a) $y[n]+y[n-1] = x[n] - x[n-1]$

$$(1+z^{-1})\,Y(z) = (1-z^{-1})\,X(z)$$

$$H(z) = \frac{1-z^{-1}}{1+z^{-1}}$$

$$\boxed{\begin{aligned} h[n] &= (-1)^n u[n] - (-1)^{n-1} u[n-1] \\ &= (-1)(-1)^{n-1} u[n] - (-1)^{n-1} u[n-1] \\ &= \delta[n] - (-1)^{n-1} u[n-1] - (-1)^{n-1} u[n-1] \\ &= \delta[n] - 2(-1)^{n-1} u[n-1] \end{aligned}}$$

b) $x[n] = n u[n] \longrightarrow X(z) = \dfrac{z^{-1}}{(1-z^{-1})^2} \qquad |z|>1$

$$Y(z) = H(z)X(z) = \frac{z^{-1}}{(1+z^{-1})(1-z^{-1})} \qquad |z|>1$$

$$= \frac{A_1}{1+z^{-1}} + \frac{A_2}{1-z^{-1}}$$

$A_1 + A_2 = 0$

$-A_1 + A_2 = 1$

$A_2 = \frac{1}{2}$ & $A_1 = -\frac{1}{2}$

$$Y(z) = \frac{-1/2}{1+z^{-1}} + \frac{1/2}{1-z^{-1}} \qquad |z|>1$$

$$\boxed{\begin{aligned} y[n] &= -\tfrac{1}{2}(-1)^n u[n] + \tfrac{1}{2} u[n] \\ &= \tfrac{1}{2}\left[(-1)^{n+1} + 1\right] u[n] = \{\underset{y[0]}{0}, 1, 0, 1, \ldots\} \end{aligned}}$$

c) With non-zero initial conditions and $x[n] = 0 \;\; \forall n \leq 0$ we have

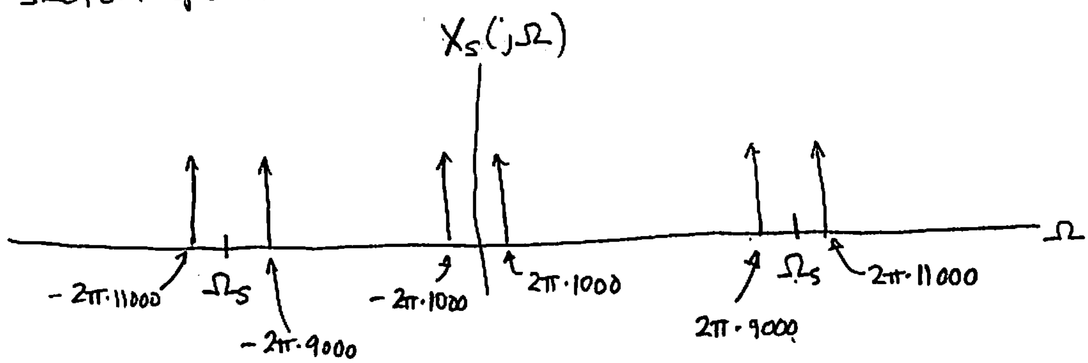
$$Y(z) = \underbrace{\frac{-y[-1]}{1+z^{-1}}}_{\text{zero input response}} + \underbrace{H(z)X(z)}_{\substack{\text{we already computed this} \\ \text{(zero initial conditions response)}}} \qquad |z|>1$$

continued...

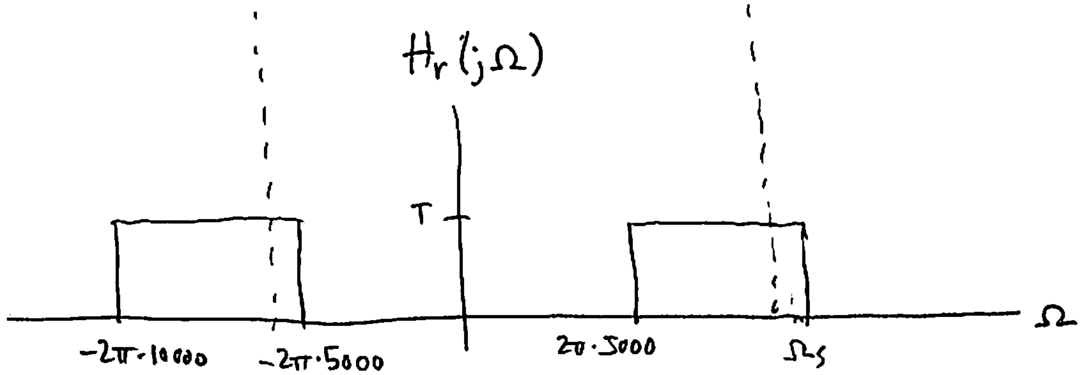
hence

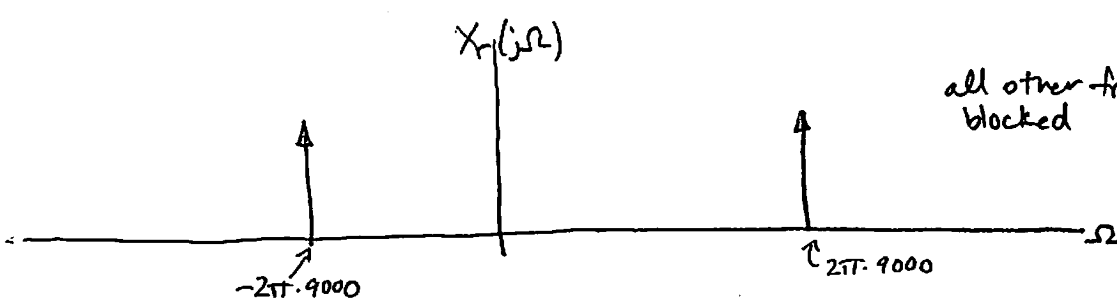
$$\boxed{\begin{aligned} y[n] &= (-1)(-1)^n u[n] + \tfrac{1}{2}\left[(-1)^{n+1}+1\right]u[n] \\ &= \left((-1)^{n+1} + \tfrac{1}{2}(-1)^{n+1} + \tfrac{1}{2}\right)u[n] \\ &= \left(\tfrac{3}{2}(-1)^{n+1} + \tfrac{1}{2}\right)u[n] \\ &= \{\underset{y[-1]}{1}, \underset{y[0]}{-1}, 2, -1, 2, \ldots\} \end{aligned}}$$

$\boxed{2.}$ Sketch spectra...



$\Omega_s = 2\cdot\pi\cdot 10^3$ rad/sec





all other frequencies blocked

Hence $\boxed{x_r(t) = \cos(2\pi\cdot 9000\, t)}$

By recovering with a bandpass filter, we have shifted the input from 1kHz to 9kHz.