ECE503 Spring 2014 Quiz 4

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 40 points. Given the spectrum of the continuous-time signal in Fig. 1 and the system in Fig. 1 with ideal continuous/discrete and discrete/continuous blocks. Suppose the sampling rate is $f_s = \frac{1}{T} = 20$ kHz and the filter $H(e^{j\omega})$ is an ideal lowpass filter with

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega + r2\pi| \leq \frac{\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

for integer r. *Neatly* sketch the spectrum of every signal in the system, making sure to label all axes as well as all relevant frequencies and amplitudes.

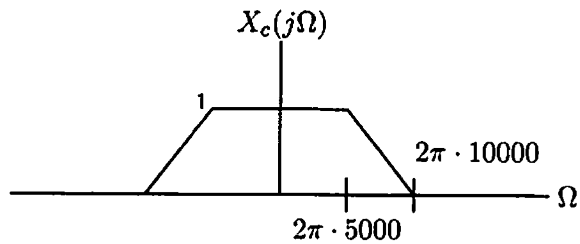


Figure 1: Spectrum of continuous-time input signal.

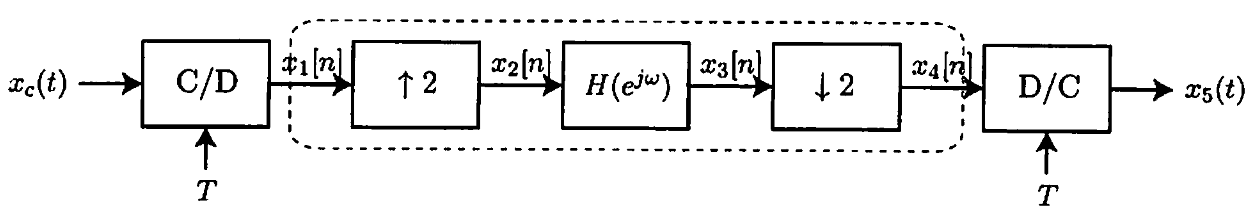
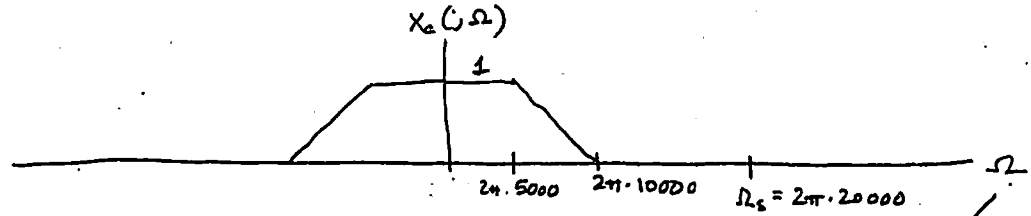


Figure 2: System for processing $x_c(t)$.

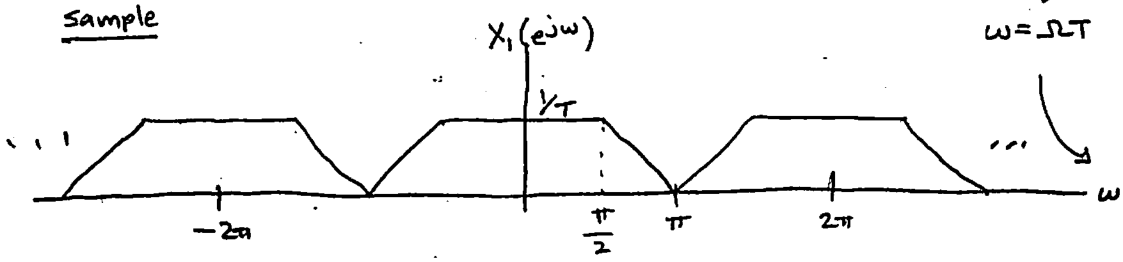
2. 30 points. Under the same conditions as problem 1, determine the minimum sampling frequency so that the overall system from $x_c(t)$ to $x_5(t)$ is LTI.

3. 30 points. Now suppose in Fig. 1 the discrete-time system is given as $H(z) = 1 + az^{-1} + bz^{-2} + cz^{-3}$ (the DT system is FIR and no longer an ideal LPF). Redesign the system from $x_1[n]$ to $x_4[n]$ so that less computation is required but the output is identical to the original system. Estimate the amount of computation saved with respect to Fig. 1.

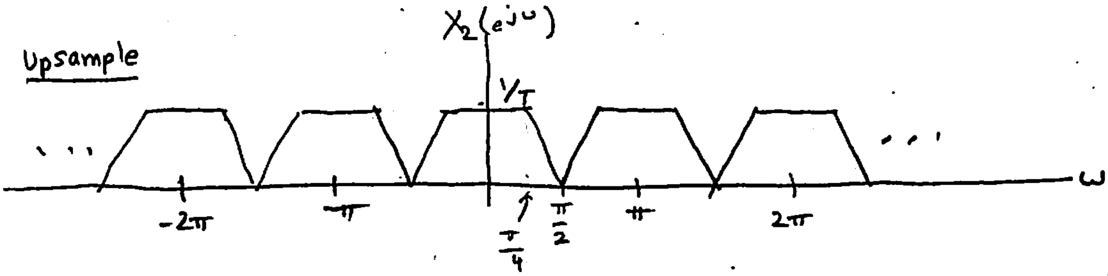
1.

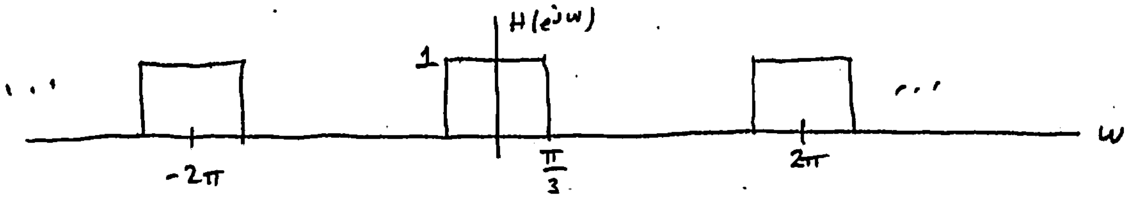


sample

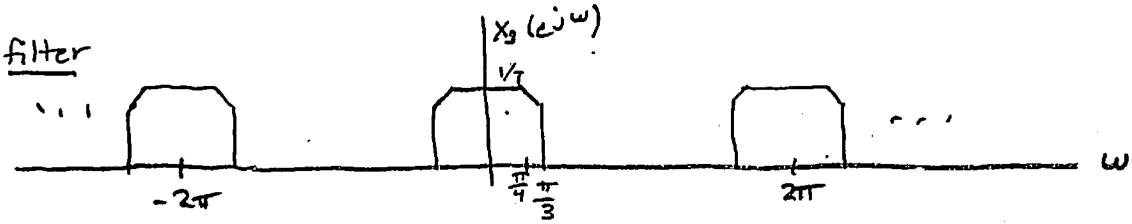


Upsample

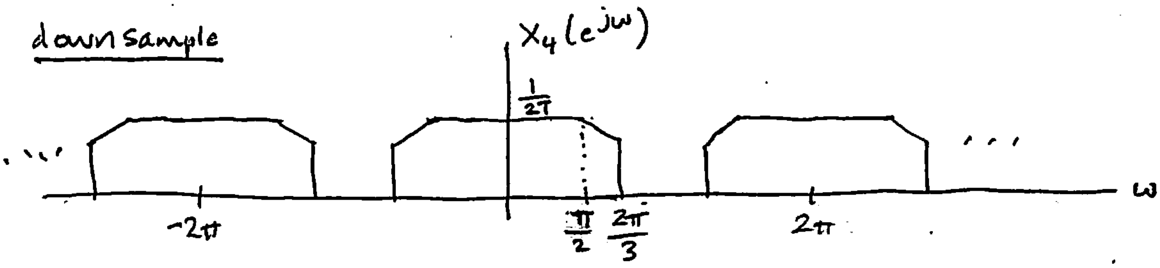




filter

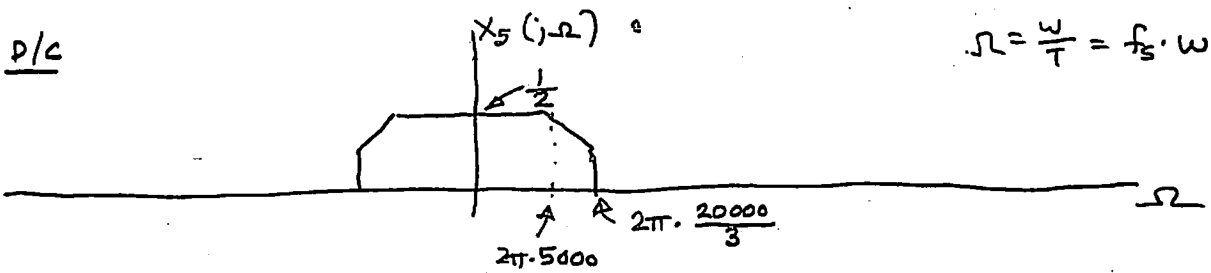


down sample



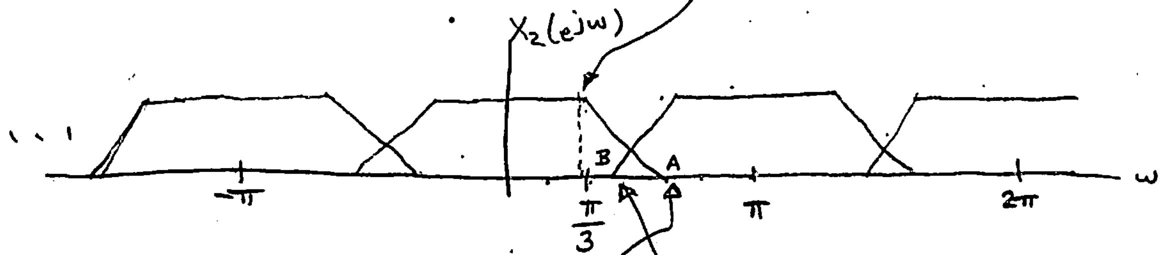
D/C

$\Omega = \frac{\omega}{T} = f_s \cdot \omega$



2. For the overall system to be LTI, any aliasing can't propagate to the output. The LPF will block some amount of aliasing.
Recall that the cutoff frequency of the DT system is $\pi/3$.

After upsampling, we have



we need this point to be $\geq \frac{\pi}{3}$

A. This point is at $2\pi \cdot 10000 \cdot \frac{T}{2}$

B. This point is at $\pi - 2\pi \cdot 10000 \cdot \frac{T}{2}$

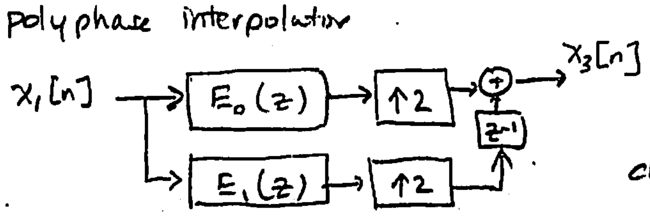
We require $\pi - 2\pi \cdot 10000 \frac{T}{2} \geq \frac{\pi}{3}$

$$\frac{\cancel{2\pi}}{3} \geq \cancel{2\pi} \cdot 10000 \frac{T}{2}$$

$$T \leq \frac{2}{3 \cdot 10000} \quad \Rightarrow \quad \boxed{f_s \geq 15{,}000}$$

3. You could do a polyphase interpolator or decimator here with the same computational savings.

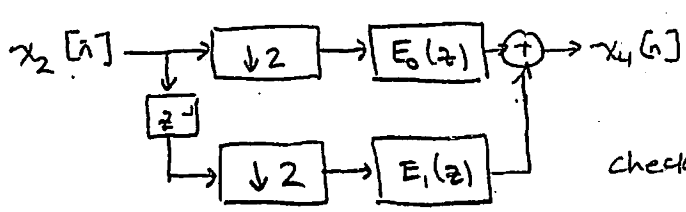
polyphase interpolator



$E_0(z) = 1 + bz^{-1}$
$E_1(z) = a + cz^{-1}$

check: $H(z) = E_0(z^2) + z^{-1} E_1(z^2)$
$= 1 + bz^{-2} + az^{-1} + cz^{-3}$ ✓

or polyphase decimator



$E_0(z) = 1 + bz^{-1}$
$E_1(z) = a + cz^{-1}$ } same

check: $H(z) = E_0(z^2) + z^{-1} E_1(z)$ ✓

The original scheme required 8 MACs per input sample (due to ↑2 before filter)
These polyphase realizations require 4 MACs per input sample. ≈ half computation