ECE503 Spring 2014 Quiz 6

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 40 points total. Suppose you have a LTI system with impulse response $h[n] = (0.9)^n u[n]$.
   (a) 20 points. Write an expression for the phase response of the system in the form of
   $$\angle H(e^{j\omega}) = \arctan\left(\frac{f(\omega)}{g(\omega)}\right)$$
   where you explicitly specify $f(\omega)$ and $g(\omega)$.
   (b) 20 points. Compute the group delay of this system as a function of ω. Hint:
   $$\frac{\partial}{\partial\omega}\arctan\left(\frac{f}{g}\right) = \frac{f'g - g'f}{f^2 + g^2}$$

2. 40 points. Suppose you have a causal LTI system with transfer function
   $$H(z) = \frac{1 - z^{-2}}{(1 - j0.9z^{-1})(1 + j0.9z^{-1})}$$
   Draw the pole-zero diagram and sketch the magnitude response of this system for $0 \leq \omega \leq \pi$. Also determine the peak magnitude response.

3. 20 points. Suppose you have a causal LTI system with transfer function
   $$H(z) = \frac{1 - \frac{1}{4}z^{-2}}{(1 - j0.9z^{-1})(1 + j0.9z^{-1})}$$
   List all causal stable rational transfer functions with the same number of poles and zeros and with the same magnitude response. Would your answer be different if you could have more poles and zeros?

1. $h[n] = (0.9)^n u[n] \rightarrow H(e^{jw}) = \dfrac{1}{1-0.9e^{-jw}}$

$$H(e^{jw}) = \frac{1-0.9e^{jw}}{(1-0.9e^{-jw})(1-0.9e^{jw})}$$

$$= \frac{1-0.9e^{jw}}{1-0.9e^{-jw}-0.9e^{jw}+0.81}$$

$$= \frac{1-0.9e^{jw}}{1.81-1.8\cos(w)}$$

denominator is real and always positive, so it doesn't affect the phase.

$$\boxed{\begin{aligned} \measuredangle H(e^{jw}) &= \arctan\left(\frac{\text{Imag}(1-0.9e^{jw})}{\text{Real}(1-0.9e^{jw})}\right) \\ &= \arctan\left(\frac{-0.9\sin w}{1-0.9\cos w}\right) \end{aligned}}$$

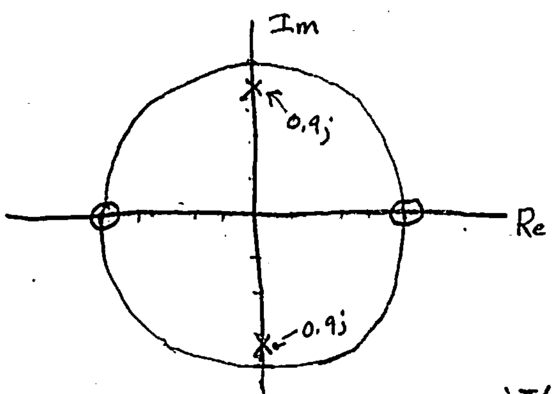
$$\tau_g(w) = -\frac{\partial}{\partial w}\measuredangle H(e^{jw})$$

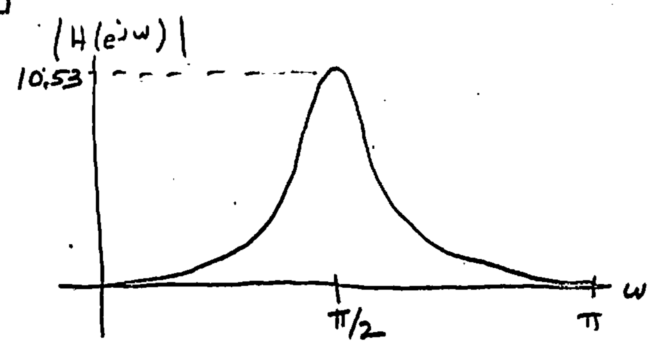
$$= \frac{g'f-f'g}{f^2+g^2} = \frac{(0.9\sin w)(-0.9\sin w)-(-0.9\cos w)(1-0.9\cos w)}{0.81\sin^2 w+(1-0.9\cos w)^2}$$

$$= \frac{-0.81(\sin^2 w+\cos^2 w)+0.9\cos w}{0.81(\sin^2 w+\cos^2 w)+1-1.8\cos w}$$

$$\boxed{\tau_g(w) = \frac{0.9\cos w-0.81}{1.81-1.8\cos w}}$$

2.

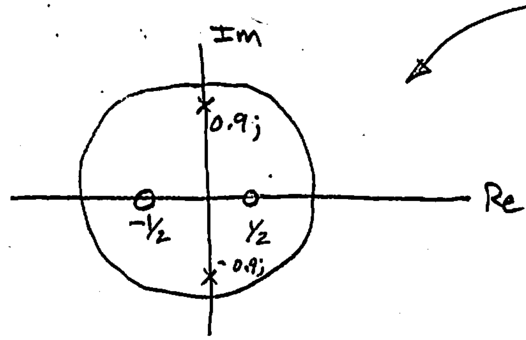




Peak magnitude occurs at $e^{j\pi/2} = j$. We can compute

$$|H(e^{jw})|_{w=\frac{\pi}{2}} = \frac{|+1-j|\cdot|-1-j|}{|0.9j-j|\cdot|-0.9j-j|} = \frac{\sqrt{2}\cdot\sqrt{2}}{(0.1)\cdot(1.9)} = \frac{2}{0.19} \approx \boxed{10.53}$$

3.

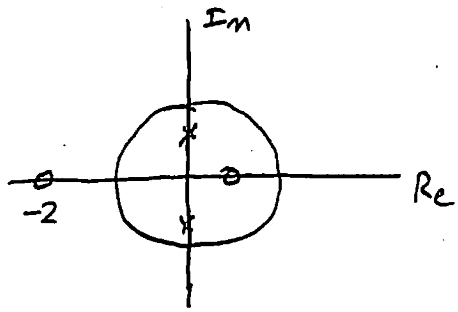


$$H(z) = \frac{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}{(1-0.9jz^{-1})(1+0.9jz^{-1})}$$

We know we can reflect each zero outside of the unit circle without changing the magnitude response (as long as we account for scaling)

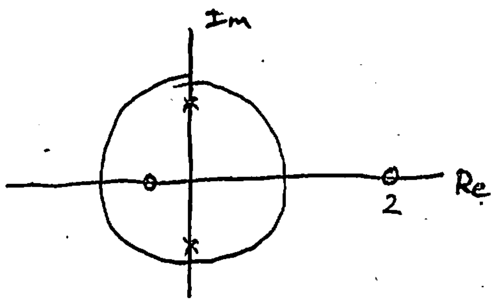
So

A.



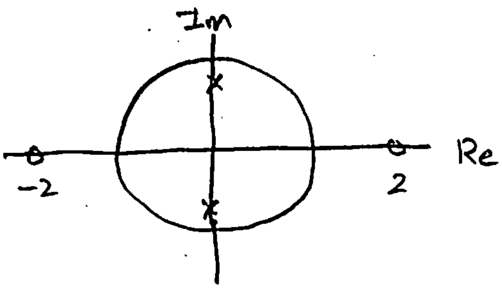
$$H_A(z) = \frac{\frac{1}{2}(1+2z^{-1})(1-\frac{1}{2}z^{-1})}{(1-0.9jz^{-1})(1+0.9jz^{-1})}$$

B.



$$H_B(z) = \frac{(1+\frac{1}{2}z^{-1})(1-2z^{-1})\frac{1}{2}}{(1-0.9jz^{-1})(1+0.9jz^{-1})}$$

C.



$$H_C(z) = \frac{(1+2z^{-1})(1-2z^{-1})\frac{1}{4}}{(1-0.9jz^{-1})(1+0.9jz^{-1})}$$

all have same magnitude response (but different phase responses)

If we were allowed to have more poles & zeros, we could cascade allpass systems without changing the magnitude response. There would be an unlimited number of such systems.