ECE503 Spring 2014 Quiz 7

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points total (6 points each). Several properties of LTI systems can be determined immediately by inspection of the pole-zero plot of a rational $H(z)$. Suppose $H(z)$ is known to be causal. List the characteristic(s) of the pole-zero plot would allow you to immediately identify that $H(z)$ has each of the following properties:

   (a) stable.

   (b) minimum phase.

   (c) all-pass.

   (d) impulse response $h[n]$ is real-valued.

   (e) impulse response $h[n]$ is finite-length.

2. 30 points. Given a causal linear-phase system with finite impulse response

$$h[0] = 1, \quad h[1] = \frac{10}{3}, \quad h[2] = 1$$

   find a causal and stable system $G(z)$ such that $G(z)H(z) = F(z)$ where $F(z)$ is an all-pass system with $|F(e^{j\omega})| = 1$ for all ω. Explicitly specify $G(z)$ and $F(z)$.

3. 40 points. Given the sub-systems

$$\begin{aligned} H_1(z) &= 1 + z^{-1}, \\ H_2(z) &= 1 - z^{-1}, \\ H_3(z) &= 1 + 2z^{-1}, \text{ and} \\ H_4(z) &= 1 + \frac{1}{2}z^{-1}, \end{aligned}$$

   form type I, II, III, and IV causal FIR generalized linear phase systems as a cascade realization of <u>two or more</u> of these subsystems. Confirm your cascaded system meets the requirements by writing out $H(z)$ or $h[n]$ explicitly for each type.

SOLUTION

1. $H(z)$ causal (given)

a) stable $\Leftrightarrow$ all poles inside the unit circle

b) minimum phase $\Leftrightarrow$ all poles and zeros inside the unit circle.

c) all pass $\Leftrightarrow$ each pole of the form $re^{j\theta}$ has a matching zero at $\frac{1}{r}e^{-j\theta}$.

d) $h[n]$ real valued $\Leftrightarrow$ all complex poles & zeros appear in conjugate pairs.

e) $h[n]$ finite length $\Leftrightarrow$ poles only at $z=0$ or $z=\infty$.

2. $$H(z) = (1+3z^{-1})(1+\tfrac{1}{3}z^{-1})$$

$$= \underbrace{(1+\tfrac{1}{3}z^{-1})^2}_{\text{minimum phase}} \underbrace{\frac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}}_{\text{allpass but magnitude response } = 3}$$

so $$H(z) = \underbrace{3(1+\tfrac{1}{3}z^{-1})^2}_{\text{still minimum phase}} \cdot \underbrace{(\tfrac{1}{3})\left(\frac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}\right)}_{\text{allpass with magnitude response } =1}$$

let $$G(z) = \frac{1}{3(1+\frac{1}{3}z^{-1})^2}$$

then $$G(z)H(z) = F(z) = \left(\tfrac{1}{3}\right)\left(\frac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}\right) \quad \text{allpass}$$

Note that $G(z)$ is inverting the minimum phase component of $H(z)$, resulting in a flat magnitude response.

3. Figure 5.38 is helpful here since it shows the zero locations for each type.

Type I: $H_I(z) = H_3(z)H_4(z) = 1+\frac{5}{2}z^{-1}+z^{-2}$

Type II: $H_{II}(z) = H_1(z)H_3(z)H_4(z) = 1+\frac{7}{2}z^{-1}+\frac{7}{2}z^{-2}+z^{-3}$

Type III: $H_{III}(z) = H_1(z)H_2(z) = 1-z^{-2}$

Type IV: $H_{IV}(z) = H_2(z)H_3(z)H_4(z) = 1+\frac{3}{2}z^{-1}-\frac{3}{2}z^{-2}-1$