# ECE503 Spring 2014 Quiz 8

Your Name: _____ ECE Box Number: _____

**Instructions**: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 45 points total. Given the realization structure shown in Fig. 1 below.
   (a) 5 points. What is the common name of this realization structure?
   (b) 20 points. Determine the transfer function $H(z) = \frac{Y(z)}{X(z)}$ of this system. Your answer will be a function of $a_1$ and $a_2$.
   (c) 20 points. Is there any choice for $\{a_1, a_2\}$ such that $H(z)$ can be realized with less than two delays? Explain.

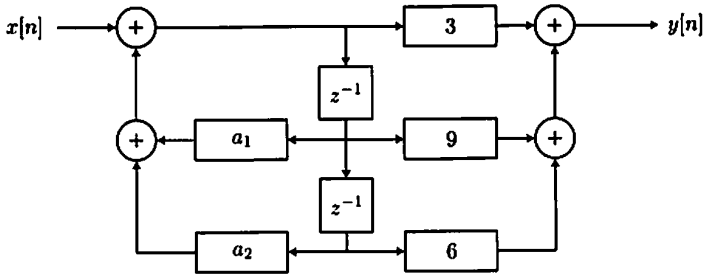



Figure 1: Realization structure.

2. 25 points. Assuming $a_1 = 5$ and $a_2 = -4$, realize the system shown in Fig. 1 in cascaded form with single-order sections using a total of two delay elements. Is your answer unique?

3. 30 points. Given the causal lattice filter shown in Fig. 2 with parameters $\{a, b, c, d\}$. Determine the transfer functions of the system $H(z) = \frac{Y(z)}{X(z)}$ and $G(z) = \frac{W(z)}{X(z)}$.

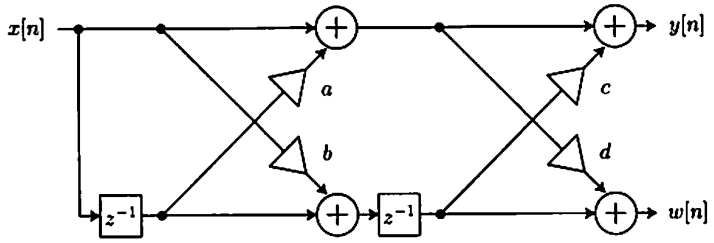



Figure 2: Causal lattice filter.

**1.** a) This is the "direct form II" realization structure.

b) Can determine $H(z)$ directly from the block diagram, taking care to get the sign of the denominator coefficients correct:

$$\boxed{H(z) = \frac{3 + 9z^{-1} + 6z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}}$$

c) If we set $a_1 = -3$ and $a_2 = -2$ then we have

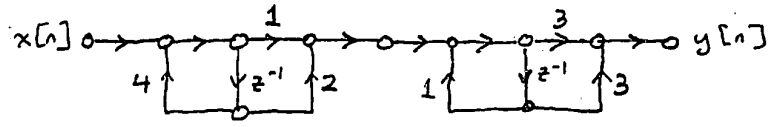
$$H(z) = \frac{3 + 9z^{-1} + 6z^{-2}}{1 + 3z^{-1} + 2z^{-2}} = 3$$

In this case $H(z)$ can be realized with <u>no delays</u> since it is a simple multiplication.

Other choices of $\{a_1, a_2\}$ could result in pole/zero cancellations as well, reducing the number of required delays.

**2.** Here we have $H(z) = \dfrac{3 + 9z^{-1} + 6z^{-2}}{1 - 5z^{-1} + 4z^{-2}} = \dfrac{3(1 + 2z^{-1})(1 + z^{-1})}{(1 - 4z^{-1})(1 - z^{-1})}$

So a first-order cascade realization could be




note that I have absorbed the 3 into the $(1+z^{-1})$ term in the numerator.

This answer is <u>not unique</u> since we could pair different numerators with different denominators and also re-order the sections (as well as consider other forms, e.g. transposed)

**3.** This is a FIR lattice. We can denote $V[n]$ as the output of the top sum and $U[n]$ as the output of the bottom sum.

$$V(z) = X(z) + az^{-1}X(z) = (1 + az^{-1})X(z) \qquad \text{(top sum)}$$

$$U(z) = bX(z) + z^{-1}X(z) = (b + z^{-1})X(z) \qquad \text{(bottom sum)}$$

$$Y(z) = V(z) + cz^{-1}U(z) = (1 + az^{-1})X(z) + cz^{-1}(b + z^{-1})X(z)$$

$$\Rightarrow \boxed{H(z) = 1 + (a + bc)z^{-1} + cz^{-2}}$$

$$W(z) = dV(z) + z^{-1}U(z) = d(1 + az^{-1})X(z) + z^{-1}(b + z^{-1})X(z)$$

$$\Rightarrow \boxed{G(z) = d + (ad + b)z^{-1} + z^{-2}}$$