ECE503 Spring 2014 Quiz 9

Your Name: ______________________________ ECE Box Number: ________

Instructions: This quiz is worth a total of 100 points. The quiz is open book and open notes. You may also use a calculator. You may not use a computer, phone, or tablet. Please show your work on each problem and box/circle your final answers. Points may be deducted for a disorderly presentation of your solution.

1. 30 points. Suppose you have a linear time invariant IIR filter with transfer function

$$H(z) = \frac{0.6 - 3.6z^{-1} + 7.8z^{-2} - 7.2z^{-3} + 2.4z^{-4}}{1 - 2.2z^{-1} + 3.07z^{-2} - 2.142z^{-3} + 0.9506z^{-4}}$$

   and you wish to implement this filter in direct form I with all filter coefficients stored as signed 6-bit fixed-point numbers. Determine the optimum number of fractional bits for the numerator and denominator coefficients (the answers may be different) to minimize the coefficient quantization error.

2. 70 points. Suppose you wish to implement the transfer function

$$H(z) = \frac{1 + bz^{-1}}{1 - az^{-1}}$$

   with a and b both real, $a \neq -b$, and $0 < a < 1$. You have two choices for implementation:

   - direct form I (as shown in Fig. 6.14) or
   - direct form II (as shown in Fig. 6.15).

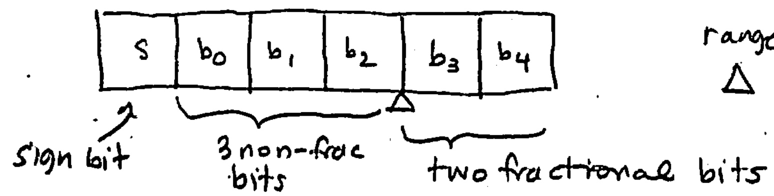
   It is assumed that the system is implemented with two's-complement fixed-point arithmetic and that all products (multiplications by one do not introduce noise) are immediately rounded before any additions are performed. Under the usual product round-off noise assumptions (each product round-off error is a zero-mean white process with variance σ_B^2 and is independent of the other product round-off noises), denote $f[n]$ as the total product round-off noise at the output. Determine the conditions on a and b for which a direct form I implementation will have less total round-off noise power σ_f^2 at the output.

1. We just need to look at the maximum numerator and denominator coefficients to determine the number of non-fractional bits needed to avoid overflow.

Numerator : max coefficient = 7.8 $\Rightarrow$ Need 3 non-frac bits

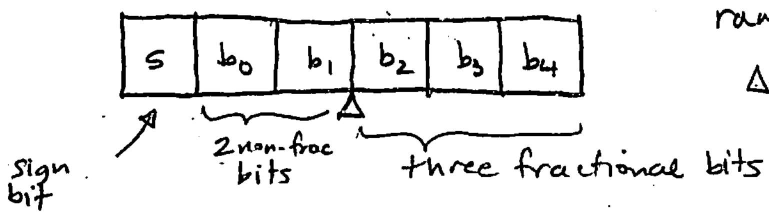
Denominator : max coefficent = 3.07 $\Rightarrow$ need 2 nonfrac bits

Numerator



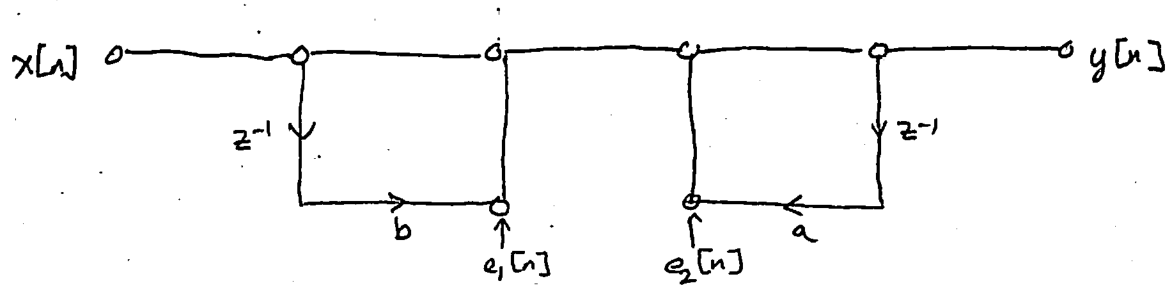
range : -8 to 7.75
Δ : 0.25

Denominator



range : -4 to 3.875
Δ : 0.125

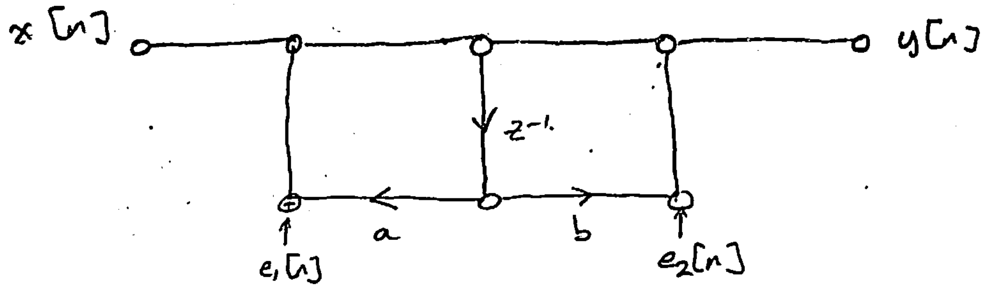
2. In direct form I, we have



Note that $e_1[n]$ and $e_2[n]$ both propagate through just the first-order system $G(z) = \frac{1}{1-az^{-1}}$

Hence, for DF-I, we have $\sigma_f^2 = 2\sigma_B^2 \frac{1}{1-a^2} = \sigma_B^2 \frac{2}{1-a^2}$

For direct form II, we have



$e_2[n]$ is directly connected to the output

$e_1[n]$ propagates through $H(z) = \frac{1+bz^{-1}}{1-az^{-1}} = \frac{z+b}{z-a} = 1 + \frac{a+b}{z-a}$

Hence, for DF-II, we have

$$\sigma_f^2 = \sigma_B^2 + \underbrace{\sigma_B^2\left(1 + \frac{(a+b)^2}{1-a^2}\right)}_{\text{via table lookup or any other method.}} = \sigma_B^2\left(2 + \frac{(a+b)^2}{1-a^2}\right)$$

$$\sigma_f^2 = \sigma_B^2\left(\frac{2-2a^2+a^2+2ab+b^2}{1-a^2}\right)$$

Hence, DF I is preferred if

$$2 < 2 - a^2 + 2ab + b^2$$

$$\Leftrightarrow \boxed{a^2 - 2ab - b^2 < 0}$$

← this makes intuitive sense since DF-I is not affected by b and the noise power for DF-II increases with b