ECE504 Homework Assignment Number 1
Due by 8:50pm on 16-Sep-2008

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 8 pts. Given the circuit in Figure 1,

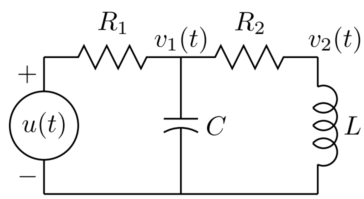

Figure 1: A circuit.

   (a) Let the state $\boldsymbol{x}(t) = [v_1(t)\ v_2(t)]^\top$ and let the output $y(t)$ be equal to the current through R_2. Find the matrices $\boldsymbol{A}(t)$, $\boldsymbol{B}(t)$, $\boldsymbol{C}(t)$, and $\boldsymbol{D}(t)$ such that

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)u(t) \\ y(t) &= \boldsymbol{C}(t)\boldsymbol{x}(t) + \boldsymbol{D}(t)u(t). \end{aligned}$$

   (b) Write the input-output differential equation for this system, e.g. $y(t) = f(\dot{y}(t), \ddot{y}(t), \dots, u(t), \dot{u}(t), \ddot{u}(t), \dots)$.

   (c) Write the transfer function for this system.

   (d) Classify this system:

      i. Memoryless, lumped, or distributed
      ii. Causal or noncausal
      iii. Linear or nonlinear
      iv. Time varying or time invariant

2. 4 pts. Chen Problem 2.6.

3. 4 pts. Qualitative description of systems:

   (a) Suppose we are given a system with two inputs and one output with an input-output relationship

$$y(t) = \min(u_1(t), u_2(t)).$$

   Classify this system as in 1.(d).

   (b) Suppose we have a discrete-time system with an input-output relationship

$$y(k) = \frac{1}{1-\lambda} \sum_{m=0}^{\infty} \lambda^m u(k-m+1)$$

   where $0 < \lambda < 1$ is known as the "forgetting factor". Classify this system as in 1.(d). If $u(k) = 1\ \forall k$, what is $y(k)$?

4. 5 pts. Suppose we have a discrete time system that computes the moving average of a finite number of past inputs such that

$$y(k) \quad = \quad \frac{1}{N}\sum_{n=0}^{N-1} u(k-n).$$

(a) Let $N = 4$ and let the state $x(k) = [u(k-1)\ u(k-2)\ u(k-3)]^\top$. Find the matrices $\boldsymbol{A}(k)$, $\boldsymbol{B}(k)$, $\boldsymbol{C}(k)$, and $\boldsymbol{D}(k)$ such that

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)u(k) \\ y(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)u(k). \end{aligned}$$

(b) Let $N = 4$ and let the state

$$x(k) \quad = \quad \begin{bmatrix} u(k-1) + u(k-2) + u(k-3) \\ u(k-1) + u(k-2) \\ u(k-1) \end{bmatrix}$$

Find the matrices $\boldsymbol{A}(k)$, $\boldsymbol{B}(k)$, $\boldsymbol{C}(k)$, and $\boldsymbol{D}(k)$ such that

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)u(k) \\ y(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)u(k). \end{aligned}$$

(c) Comment on the uniqueness of the state.

5. 4 pts. Find the transfer function $\hat{g}(s) = \frac{\hat{y}(s)}{\hat{u}(s)}$ of the system

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} c_1 & c_2 \end{bmatrix} \boldsymbol{x}(t) + du(t). \end{aligned}$$

Simplify your answer to the form

$$\hat{g}(s) = \frac{\lambda_2 s^2 + \lambda_1 s + \lambda_0}{\gamma_2 s^2 + \gamma_1 s + \gamma_0}.$$

Hint: Recall that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \quad = \quad \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

6. 5 pts. Suppose you are given the 2-dimensional system described by the state space equations

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \end{bmatrix} u(t). \end{aligned}$$

Use Matlab to plot the impulse response of this system (relaxed initial conditions). Now suppose you are given the 1-dimensional system described by the state space equations

$$\begin{aligned} \dot{x}(t) &= -2x(t) + u(t) \\ y(t) &= x(t). \end{aligned}$$

Again, use Matlab to plot the impulse response of this system (relaxed initial conditions). Compare your results to the 2-dimensional system. What is going on? Explain analytically.