ECE504 Homework Assignment Number 5
Due by 8:50pm on 18-Nov-2008

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 3 pts. Chen 5.4

2. 3 pts. Chen 5.7

3. 4 pts. Suppose you are given a discrete time system with

$$\begin{aligned} \boldsymbol{x}[k+1] &= \begin{bmatrix} 0.9 & 1 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}[k] + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u[k] \\ y[k] &= \begin{bmatrix} c_1 & c_2 \end{bmatrix} \boldsymbol{x}[k] \end{aligned}$$

   Find necessary and sufficient conditions on the $\boldsymbol{B}$ and $\boldsymbol{C}$ matrices such that this system is BIBO stable.

4. 4 pts. Suppose you are given a discrete time system with

$$\boldsymbol{x}[k+1] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \boldsymbol{x}[k] + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u[k]$$

   Find the set of reachable states, parameterized by b_1 and b_2. Find the set of controllable states, parameterized by b_1 and b_2. Do there exist values for b_1 and b_2 such that the set of reachable states is different from the set of controllable states?

5. 4 pts. Suppose you have an LTI discrete time system with reachable states $\bar{\boldsymbol{x}} \in \mathbb{R}^n$ and $\tilde{\boldsymbol{x}} \in \mathbb{R}^n$. Prove that there exists an an input $\{u[k]\}_{k=0}^{n-1}$ such that the state $\boldsymbol{x}[n] = \tilde{\boldsymbol{x}}$ given the initial state $\boldsymbol{x}[0] = \bar{\boldsymbol{x}}$.

6. 4 pts. Suppose

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 0 & 1 & -1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \boldsymbol{x}(t). \end{aligned}$$

   Find bases for the reachable and unobservable subspaces of this system in $\mathbb{R}^3$. Is this a "reachable system"? Is this an "observable system"?

7. 4 pts. Suppose $\boldsymbol{C}e^{t\boldsymbol{A}}\boldsymbol{B} = \bar{\boldsymbol{C}}e^{t\boldsymbol{A}}\boldsymbol{B}$ for every $t \geq 0$ and that $\boldsymbol{A}$ and $\boldsymbol{B}$ are such that the continuous time LTI state-space system defined by $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ is reachable. Prove that $\boldsymbol{C}$ must be equal to $\bar{\boldsymbol{C}}$. Hint: This is a proof that $\boldsymbol{C}$ is *unique*. Think about some of the methods we used to prove uniqueness earlier in this course. Also consider all of the criteria available to you regarding the reachability of systems described lecture and in the Chen textbook.

8. 4 pts. Suppose $n > 1$ and let a nonzero $\boldsymbol{v} \in \mathbb{R}^n$ be given. Describe bases for the reachable and unobservable subspaces of

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= -\boldsymbol{v}\boldsymbol{v}^\top \boldsymbol{x}(t) + \boldsymbol{v}u(t) \\ y(t) &= \boldsymbol{v}^\top \boldsymbol{x}(t). \end{aligned}$$