ECE504 Homework Assignment Number 6
Due by 8:50pm on 02-Dec-2008

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 3 pts. For LTI continuous time state-space systems, prove that the state $\bar{\boldsymbol{x}}$ is unobservable if and only if $\bar{\boldsymbol{x}} \in \text{range}(\boldsymbol{Q}_o)$.

2. 3 pts. For LTI continuous time state-space systems, prove that if the system $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ is not observable then $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ is not a minimal realization.

3. 3 pts. Chen 7.12. Note that if two systems $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ and $\{\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}, \bar{\boldsymbol{C}}, \bar{\boldsymbol{D}}\}$ are algebraically equivalent then there exists some invertible $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ such that

$$\begin{aligned} \bar{\boldsymbol{A}} &= \boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P} \\ \bar{\boldsymbol{B}} &= \boldsymbol{P}^{-1}\boldsymbol{B} \\ \bar{\boldsymbol{C}} &= \boldsymbol{C}\boldsymbol{P} \\ \bar{\boldsymbol{D}} &= \boldsymbol{D}. \end{aligned}$$

   If the two systems in this problem are algebraically equivalent then find the $\boldsymbol{P} \in \mathbb{R}^{2\times 2}$ such that the equations above hold.

4. 6 pts. Given the SISO transfer function

$$\hat{g}(s) = \frac{1}{s^3 + 1}$$

   (a) Find a minimal realization.
   (b) Find a realization that is observable but not reachable/controllable.
   (c) Find a realization that is reachable/controllable but not observable.

5. 6 pts. Suppose $n > 1$ and let a nonzero $\boldsymbol{v} \in \mathbb{R}^n$ be given. Suppose you are given a state space system

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= -\boldsymbol{v}\boldsymbol{v}^\top \boldsymbol{x}(t) + \boldsymbol{v}u(t) \\ y(t) &= \boldsymbol{v}^\top \boldsymbol{x}(t). \end{aligned}$$

   (a) Is this a minimal realization? If not, find a minimal realization .
   (b) Is the original system asymptotically stable?
   (c) Is the minimal system asymptotically stable?

6. 3 pts. For the system with transfer function

$$\hat{G}(s) = \begin{bmatrix} \frac{s-1}{s} & 0 & \frac{s-2}{s+2} \\ 0 & \frac{s+1}{s} & 0 \end{bmatrix}$$

   determine the McMillan degree of $\hat{G}(s)$ and find a *minimal* realization $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ for this system. You may want to verify your results with Matlab function `minreal`.

7. 3 pts. Chen 8.4. Solve this problem analytically but verify your results with Matlab function `place`.

8. 3 pts. Chen 8.10.