Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 3 pts. For LTI continuous time state-space systems, prove that the state $\bar{x}$ is unobservable if and only if $\bar{x} \in \text{range}(Q_o)$.

2. 3 pts. For LTI continuous time state-space systems, prove that if the system $\{A, B, C, D\}$ is not observable then $\{A, B, C, D\}$ is not a minimal realization.

3. 3 pts. Chen 7.12. Note that if two systems $\{A, B, C, D\}$ and $\{\bar{A}, \bar{B}, \bar{C}, \bar{D}\}$ are algebraically equivalent then there exists some invertible $P \in \mathbb{R}^{n \times n}$ such that

\[
\begin{align*}
\bar{A} &= P^{-1}AP \\
\bar{B} &= P^{-1}B \\
\bar{C} &= CP \\
\bar{D} &= D.
\end{align*}
\]

If the two systems in this problem are algebraically equivalent then find the $P \in \mathbb{R}^{2 \times 2}$ such that the equations above hold.

4. 6 pts. Given the SISO transfer function

\[ \hat{g}(s) = \frac{1}{s^3 + 1} \]

(a) Find a minimal realization.

(b) Find a realization that is observable but not reachable/controllable.

(c) Find a realization that is reachable/controllable but not observable.

5. 6 pts. Suppose $n > 1$ and let a nonzero $v \in \mathbb{R}^n$ be given. Suppose you are given a state space system

\[
\begin{align*}
\dot{x}(t) &= -vv^T x(t) + vu(t) \\
y(t) &= v^T x(t).
\end{align*}
\]

(a) Is this a minimal realization? If not, find a minimal realization.

(b) Is the original system asymptotically stable?

(c) Is the minimal system asymptotically stable?

6. 3 pts. For the system with transfer function

\[
\hat{G}(s) = \begin{bmatrix}
\frac{s-1}{s} & 0 & \frac{s-2}{s+2} \\
0 & \frac{s+1}{s} & 0
\end{bmatrix}
\]

determine the McMillan degree of $\hat{G}(s)$ and find a minimal realization $\{A, B, C, D\}$ for this system. You may want to verify your results with Matlab function `minreal`.

7. 3 pts. Chen 8.4. Solve this problem analytically but verify your results with Matlab function `place`.

8. 3 pts. Chen 8.10.