ECE504: Lecture 10

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Lecture 11 Major Topics

We are finishing up Part II of ECE504: **Quantitative and qualitative analysis of systems**

mathematical description $\rightarrow$ results about behavior of system

and starting Part III of ECE504:

design and control of systems

Today, we will cover:

- How to construct minimal realizations
- State feedback to stabilize and control system behavior

You should be reading Chen Chapters 7 (you can skip 7.4 and 7.5) and 8 now.

Constructing Minimal Realizations

From the proof of the Fundamental Theorem of Realization Theory, we now have a procedure to construct a minimal realization:

1. Compute $\boldsymbol{Q}_r$ for the n-state realization $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$.
   1.1 If $\operatorname{rank}(\boldsymbol{Q}_r) = n$ then the system is already reachable. Let $r = n$ and $\tilde{\boldsymbol{A}} = \boldsymbol{A}$, $\tilde{\boldsymbol{B}} = \boldsymbol{B}$, $\tilde{\boldsymbol{C}} = \boldsymbol{C}$, and $\tilde{\boldsymbol{D}} = \boldsymbol{D}$.
   1.2 Otherwise, form a basis $\boldsymbol{v}_1, \dots, \boldsymbol{v}_r$ for the range of $\boldsymbol{Q}_r$ and follow the steps in the proof to get a smaller r-state realization that is reachable. Call this realization $\{\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{C}}, \tilde{\boldsymbol{D}}\}$.
2. Now compute $\boldsymbol{Q}_o$ for the r-state realization $\{\tilde{\boldsymbol{A}}, \tilde{\boldsymbol{B}}, \tilde{\boldsymbol{C}}, \tilde{\boldsymbol{D}}\}$.
   2.1 If $\operatorname{rank}(\boldsymbol{Q}_o) = r$ then the system is already observable. Let $s = r$ and $\bar{\boldsymbol{A}} = \tilde{\boldsymbol{A}}$, $\bar{\boldsymbol{B}} = \tilde{\boldsymbol{B}}$, $\bar{\boldsymbol{C}} = \tilde{\boldsymbol{C}}$, and $\bar{\boldsymbol{D}} = \tilde{\boldsymbol{D}}$
   2.2 Otherwise, form a basis $\boldsymbol{w}_1, \dots, \boldsymbol{w}_s$ for the nullspace of $\boldsymbol{Q}_o$ and follow the steps in the proof to get a smaller s-state realization $\{\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}, \bar{\boldsymbol{C}}, \bar{\boldsymbol{D}}\}$.
3. The s-state realization $\{\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}, \bar{\boldsymbol{C}}, \bar{\boldsymbol{D}}\}$ will be both reachable and observable, so this system will be minimal.

See Chen Chapter 7 for other approaches to finding minimal realizations.

Single-Input Single-Output Systems

Special case $p = q = 1$.

$$\hat{g}(s) = \underbrace{\frac{N(s)}{D(s)}}_{\text{proper}} = \underbrace{\frac{\hat{N}(s)}{D(s)}}_{\text{strictly proper}} + \underbrace{D}_{\text{constant}}$$

Recall that we found one realization for systems of this type a while back:

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \begin{bmatrix} -a_{n-1} & -a_{n-2} & \dots & -a_1 & -a_0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 0 & \dots & 0 & b_m & \dots & b_0 \end{bmatrix} \boldsymbol{x}(t) + Du(t)
\end{aligned}$$

This realization actually has a special name. It is called the **controllable canonical form**. Is this system minimal? How can we check?

SISO Systems: Controllable Canonical Form

First check reachability/controllability of the controllable canonical form by computing $\boldsymbol{Q}_r$ and looking at its range...

SISO Systems: Controllable Canonical Form

So we know that the controllable canonical form realization must be reachable/controllable. What about observable?

Checking the observability by computing $\boldsymbol{Q}_o$ and looking at its range is a bit tougher. Instead, we can use the following theorem.

Theorem (Chen Theorem 7.1)

The controllable canonical form is observable if and only if $\hat{N}(s)$ and $D(s)$ are coprime.

See the proof in your textbook.

Definition

The polynomials $p_1(s)$ and $p_2(s)$ are **coprime** if they share no common roots.

SISO Systems: Observable Canonical Form

Another realization for SISO systems can be found by setting $\boldsymbol{A} = \boldsymbol{A}^\top$, $\boldsymbol{B} = \boldsymbol{C}^\top$, $\boldsymbol{C} = \boldsymbol{B}^\top$, and $D = D$:

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \begin{bmatrix} -a_{n-1} & 1 & 0 & \dots & 0 \\ -a_{n-2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & 0 & 0 & \dots & 1 \\ -a_0 & 0 & 0 & \dots & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_m \\ \vdots \\ b_0 \end{bmatrix} u(t) \\
y(t) &= \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix} \boldsymbol{x}(t) + Du(t)
\end{aligned}
$$

This realization is called the "observable canonical form". It is easy to check that this system is always observable by computing $\boldsymbol{Q}_o$ and confirming its range is all of $\mathbb{R}^n$.

The observable canonical form is controllable if and only if $\hat{N}(s)$ and $D(s)$ are coprime. This can be seen as a dual to Chen Theorem 7.1.

Equivalence Transformation

Definition (Chen Definition 4.1)

Let $\boldsymbol{P} \in \mathbb{R}^{n \times n}$ be non-singular and let $\bar{\boldsymbol{x}}(t) = \boldsymbol{P}\boldsymbol{x}(t)$. Then the state-space description

$$\begin{aligned} \dot{\bar{\boldsymbol{x}}}(t) &= \bar{\boldsymbol{A}}\bar{\boldsymbol{x}}(t) + \bar{\boldsymbol{B}}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \bar{\boldsymbol{C}}\bar{\boldsymbol{x}}(t) + \bar{\boldsymbol{D}}\boldsymbol{u}(t) \end{aligned}$$

where

$$\bar{\boldsymbol{A}} = \boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^{-1} \qquad \bar{\boldsymbol{B}} = \boldsymbol{P}\boldsymbol{B} \qquad \bar{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{P}^{-1} \qquad \bar{\boldsymbol{D}} = \boldsymbol{D}$$

is said to be (algebraically) equivalent to the state-space description described by $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ and $\bar{\boldsymbol{x}}(t) = \boldsymbol{P}\boldsymbol{x}(t)$ is called an "equivalence transformation".

Equivalent Minimal Systems

Theorem (Chen Theorem 7.3)

All minimal realizations of the SISO transfer function $\hat{g}(s)$ are (algebraically) equivalent.

In other words, if you are given $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ and $\{\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}, \bar{\boldsymbol{C}}, \bar{\boldsymbol{D}}\}$ such that both SS descriptions have the same TF $\hat{g}(s)$ and both are minimal, then there exists a $\boldsymbol{P}$ such that

$$\bar{\boldsymbol{A}} = \boldsymbol{P}\boldsymbol{A}\boldsymbol{P}^{-1} \qquad \bar{\boldsymbol{B}} = \boldsymbol{P}\boldsymbol{B} \qquad \bar{\boldsymbol{C}} = \boldsymbol{C}\boldsymbol{P}^{-1} \qquad \bar{\boldsymbol{D}} = \boldsymbol{D}$$

See the proof in your textbook. It is constructive in the sense that it shows you how to find $\boldsymbol{P}$ in terms of the observability and reachability matrices.

SISO Systems: Summary

$$\hat{g}(s) = \frac{\hat{N}(s)}{D(s)} + D$$

If $\hat{N}(s)$ and $D(s)$ are coprime with $\deg(D(s)) = n$ and the realization $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ has n states, then the realization is minimal.

The intuition here is that the number of states in the system must be equal to the degree of the denominator of the transfer function after all cancellations with the numerator for the system to be minimal.

In other words, you know your realization of a SISO system is minimal if $\boldsymbol{A} \in \mathbb{R}^{n \times n}$, $\hat{N}(s)$ and $D(s)$ are coprime, and $\deg(D(s)) = n$.

Multi-Input Multi-Output Systems

Recall that p-input q-output systems have a transfer function matrix $\hat{\boldsymbol{G}}(s)$ that describes the the input-output relationship between each input and each output. Example:

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{2}{s+1} & \frac{1}{s(s+1)} \end{bmatrix}$$

Realizations for CT-LTI p-input q-output systems will look like

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{aligned}$$

where $\boldsymbol{u}(t) : \mathbb{R} \mapsto \mathbb{R}^p$ and $\boldsymbol{y}(t) : \mathbb{R} \mapsto \mathbb{R}^q$ are now vectors.

The transfer function matrix $\hat{\boldsymbol{G}}(s)$ no longer has one denominator. Can we apply our intuition about minimal SISO systems to MIMO systems?

Matrix Minors

Definition

A **minor** of order r of a matrix $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ is the determinant of a smaller matrix formed by extracting r columns and r rows from $\boldsymbol{A}$.

- Note that an $n \times n$ matrix can have minors of order up to $r = n$.
- Minors are related to cofactors and the computation of the adjoint.

Example: List all of the minors of the two-input, two-output transfer matrix

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{2}{s+1} & \frac{1}{s(s+1)} \end{bmatrix}$$

Multi-Input Multi-Output Systems

Definition (Chen Definition 7.1)

The **characteristic polynomial** of a proper rational transfer matrix $\hat{\boldsymbol{G}}(s)$ is defined as the least common denominator of all minors of $\hat{\boldsymbol{G}}(s)$.

Note that it is assumed that each entry in $\hat{\boldsymbol{G}}(s)$ is coprime.

Example: What is the characteristic polynomial of the two-input, two-output transfer matrix

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{2}{s+1} & \frac{1}{s(s+1)} \end{bmatrix}?$$

What is the degree of the characteristic polynomial?

Multi-Input Multi-Output Systems

A bigger example: What is the characteristic polynomial of the three-input, two-output transfer matrix

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \frac{s}{s+1} & \frac{1}{(s+1)(s+2)} & \frac{1}{s+3} \\ \frac{-1}{s+1} & \frac{1}{(s+1)(s+2)} & \frac{1}{s} \end{bmatrix}?$$

What is the degree of the characteristic polynomial?

Minimal Realizations of MIMO Systems

Theorem (Chen Theorem 7.M2)

The degree of the characteristic polynomial $\hat{\boldsymbol{G}}(s)$ is equal to the McMillan degree of the system.

See the proof in your textbook.

One method for finding a minimal realization of a MIMO system:

1. Generate a realization of uncoupled state update and output equations.
2. Reduce the realization using the tricks we learned from the proof of the Fundamental Theorem of Realization Theory.
3. You know you are done when the matrix $\boldsymbol{A}$ has dimension equal to the degree of the characteristic polynomial of $\hat{\boldsymbol{G}}(s)$.

Example: Minimal Realization of a MIMO System

$$\hat{\boldsymbol{G}}(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ \frac{2}{s+1} & \frac{1}{s(s+1)} \end{bmatrix}$$

Finding Minimal Realizations of MIMO Systems

Some other methods that you may want to look into:

1. MIMO versions of controller canonical form and observer canonical form (may still require reduction to make the realization minimal). See Chen Section 4.4.
2. Matrix fraction decomposition (see Brogan Section 12.6)
3. Matlab function `minreal`
4. More methods discussed Chen Sections 7.7-7.9

Equivalent Minimal Systems (Matrix TF Case)

Theorem (Chen Theorem 7.M3)

All minimal realizations of the MIMO transfer function matrix $\hat{\boldsymbol{G}}(s)$ are (algebraically) equivalent.

Same idea as SISO case...

See the proof in your textbook. It is more involved than the SISO case, but it is also constructive in the sense that it shows you how to find $\boldsymbol{P}$ in terms of the observability and reachability matrices.

The main difference in the SISO and MIMO proofs is that you can no longer invert $\boldsymbol{Q}_r$ and $\boldsymbol{Q}_o$ because these matrices are not square. You can however, perform a **pseudoinverse** (Matlab function `pinv`).

Final Remarks on Realization Theory

Remark 1: If you are given $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ and asked "is this system reachable?" and/or "is this system observable?", it might be easier to just check to see if the system is minimal by seeing if the degree of the characteristic polynomial (MIMO systems) or the denominator (SISO systems) is equal to n. Recall that

minimal realization $\Leftrightarrow$ reachable and observable system

Remark 2: Realization theory for DT-LTI systems is exactly the same as CT-LTI systems with the caveat that you must express all DT transfer functions in positive powers of z.

Remark 3: We only covered the highlights of realization theory in lecture. You should definitely read Chen Sections 7.1-7.3, 7.6-7.9, and 7.11.

State Feedback: Problem Setup

Unless otherwise stated, we will assume that we have a CT-LTI system.

Problem statement:

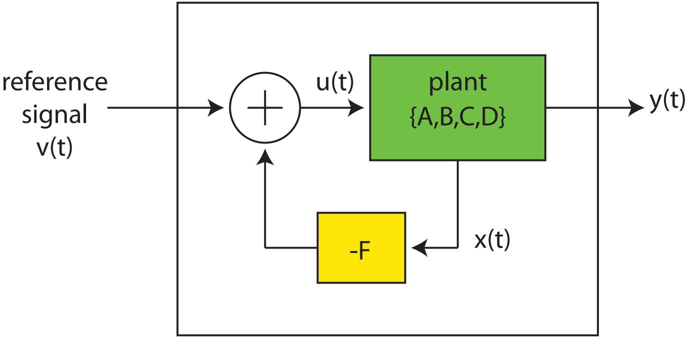
- We consider the system $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ fixed. The "uncontrolled" system is the usual

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\
\boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)
\end{aligned}
$$

- We assume that there is some exogenous function $\boldsymbol{v}(t)$ that we know.
- We want to apply an input $\boldsymbol{u}(t)$ to make the system do something, e.g.
  - make $\boldsymbol{y}(t)$ track $\boldsymbol{v}(t)$ as closely as possible (reference tracking)
  - make $\boldsymbol{y}(t)$ unaffected by $\boldsymbol{v}(t)$ (disturbance rejection)
  - make the system stable (stabilization)
  - etc.
- We require the input that we generate to be causal.
- We assume that we can measure the current state of the system.

State Feedback: Problem Setup

We consider generating inputs according to the following block diagram:



Hence

$$\boldsymbol{u}(t) \quad = \quad -\boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{v}(t)$$

for $t \in \mathbb{R}$ and $\boldsymbol{F} \in \mathbb{R}^{q \times n}$. Note that $\boldsymbol{u}(t)$ is causal because $\boldsymbol{x}(t)$ summarizes everything you need to know about the system up to time t.

How Does State Feedback Affect the System?

The input to the "system" is now $\boldsymbol{v}(t)$ and the output of the system is the usual $\boldsymbol{y}(t)$. Plugging

$$\boldsymbol{u}(t) = -\boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{v}(t)$$

into our regular (uncontrolled) system, i.e. the "plant", we get the "controlled system"

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}(-\boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{v}(t)) = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{F})\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{v}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}(-\boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{v}(t)) = (\boldsymbol{C} - \boldsymbol{D}\boldsymbol{F})\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{v}(t)\end{aligned}$$

Application: System Stabilization

- Suppose $\boldsymbol{A}$ is not Hurwitz. We know that the original uncontrolled system is not asymptotically stable. It is also not BIBO stable if this is a minimal realization.
- Can we find an $\boldsymbol{F}$ such that the controlled system is stabilized, i.e. can we find an $\boldsymbol{F}$ such that $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ is Hurwitz?

Theorem (Wonham's eigenvalue assignment theorem (1968))

Given $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ and $\boldsymbol{B} \in \mathbb{R}^{n \times p}$, there exists $\boldsymbol{F} \in \mathbb{R}^{p \times n}$ such that the n eigenvalues of $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ can be assigned to **arbitrary** *real or complex conjugate locations if and only if the original uncontrolled system $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ is reachable.*

Recall that the system $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ is reachable if and only if $\mathrm{range}(\boldsymbol{Q}_r) = \mathbb{R}^n$.

Wonham's Eigenvalue Assignment Theorem

Proof sketch of "reachability $\Rightarrow$ arbitrary e-values for $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$".

Some Remarks

1. Reachability of a system is not affected by state feedback, i.e. if $\boldsymbol{A}$ and $\boldsymbol{B}$ are reachable then so are $\bar{\boldsymbol{A}}$ and $\boldsymbol{B}$. See Chen Theorem 8.1.
2. Observability of a system may be affected by state feedback. See Chen example 8.1.
3. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are not reachable, it may still be possible to achieve some eigenvalue combinations in $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ but not all.
4. The eigenvalues of the system are not physically changed by state feedback. The feedback control law

$$\boldsymbol{u}(t) \quad = \quad -\boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{v}(t)$$

generates an input that makes the overall system (between $\boldsymbol{v}(t)$ and $\boldsymbol{y}(t)$) behave as if its eigenvalues are at different locations.

How to Find $\boldsymbol{F}$

Given a reachable system with $\boldsymbol{A}$, $\boldsymbol{B}$, and a set of desired eigenvalues, how can we find $\boldsymbol{F}$ so that $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ has the desired eigenvalues?

Four methods:

1. Matlab `place` and `acker` functions.
2. Direct method (useful for simple cases like $n = 2$ and $n = 3$).
3. Use controllable canonical form.
4. Use Lyapunov equation (Chen procedures 8.1 and 8.M1).

How to Find $\boldsymbol{F}$: Direct Method Example

Suppose we have

$$\boldsymbol{A} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

This system is clearly not stable. Is it reachable?

Let's see if we can find an $\boldsymbol{F}$ that places the eigenvalues of $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ at $\lambda_1 = -1 - j$ and $\lambda_2 = -1 + j$.

Remark: This approach results in simultaneous linear equations when $p = 1$ but results in simultaneous nonlinear equations (hard!) when $p > 1$.

How to Find $\boldsymbol{F}$: Controllable Canonical Form (1 of 2)

Procedure for single-input $p = 1$ case:

1. Given realization $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$, compute the characteristic polynomial

$$\det(\lambda \boldsymbol{I}_n - \boldsymbol{A}) = \lambda^n + q_{n-1}\lambda^{n-1} + \cdots + q_0.$$

2. Write system in controllable canonical form $\{\boldsymbol{A}_c, \boldsymbol{B}_c, \boldsymbol{C}_c, \boldsymbol{D}_c\}$.
3. Find an invertible $\boldsymbol{P} \in \mathbb{R}^{n \times n}$ such that $\boldsymbol{A}_c = \boldsymbol{P}^{-1}\boldsymbol{A}\boldsymbol{P}$, $\boldsymbol{B}_c = \boldsymbol{P}^{1}\boldsymbol{B}$, $\boldsymbol{C}_c = \boldsymbol{C}\boldsymbol{P}$, and $\boldsymbol{D}_c = \boldsymbol{D}$.
4. Using the desired eigenvalues $\{\lambda_1, \dots, \lambda_n\}$, write the desired characteristic polynomial

$$(\lambda - \lambda_1) \cdots (\lambda - \lambda_n) = \lambda^n + \alpha_{n-1}\lambda^{n-1} + \cdots + \alpha_0.$$

How to Find $\boldsymbol{F}$: Controllable Canonical Form (2 of 2)

5. Since

$$\det(\lambda \boldsymbol{I}_n - (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{G})) = \lambda^n + (q_{n-1} + g_{n-1})\lambda^{n-1} + \cdots + (q_0 + g_0)$$

   set $g_j = \alpha_j - q_j$ for all $j = 0, \ldots, n-1$.
6. Set $\boldsymbol{G} = [g_{n-1}, \ldots, g_0]$.
7. Set $\boldsymbol{F} = \boldsymbol{G}\boldsymbol{P}^{-1}$.
8. Confirm that the eigenvalues of $\bar{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ are at the correct locations.

Remark: Can be extended to $p > 1$ case (see Antsakalis textbook).

State-Feedback Examples

Conclusions

Today we covered:

- Procedures to construct minimal realizations
- Minimal realizations for SISO and MIMO systems
- State feedback to stabilize an unstable system

Next time:

- More applications of state feedback.
- What can we do if we can't measure the current state?
- State estimators.
- State feedback with estimated states.