ECE504 Final Exam

15-Dec-2009

Notes:

- This exam is worth 500 points and is to be completed in 150 minutes.
- Look over all the questions before starting.
- Budget your time to allow enough time to work on each question.
- To receive maximum credit, you must show your reasoning and/or work.

1. 80 points. Given a DT-LTI system with

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

find an input sequence $\{u[0], u[1]\}$ that drives the system from the initial state $\boldsymbol{x}[0] = [1, 0]^\top$ to the final state $\boldsymbol{x}[2] = [3, 1]^\top$. If such an input sequence does not exist, explain why. If an input sequence does exist, determine if it is unique.

2. 120 points total. Given a CT-LTI state-space system described by

$$\boldsymbol{A} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \quad \boldsymbol{D} = 0,$$

   (a) 30 points. Is this $\boldsymbol{A}$ matrix diagonalizable? Why or why not? Hint: There are two distinct eigenvalues of $\boldsymbol{A}$ at $\lambda_1 = +j$ and $\lambda_2 = -j$.

   (b) 30 points. Compute the transfer function of this system. Hint: You don't need to perform a 4×4 matrix inverse to find the transfer function here. Think about the *input-output* behavior of this system.

   (c) 30 points. Discuss the stability properties of this system.

   (d) 30 points. Find a minimal realization for this system.

3. 100 points. For the MIMO system with transfer function matrix

$$\hat{G}(s) = \begin{bmatrix} 1/s & 1/s \\ 0 & 1/s \end{bmatrix}$$

determine the McMillan degree of $\hat{G}(s)$ and find any valid realization $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ for this system. Your realization doesn't have to be minimal.

4. 120 points total. Given the CT-LTI state-space system described by

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad \boldsymbol{D} = 0,$$

   (a) 20 points. Using a state feedback control rule $\boldsymbol{F}$, it it possible to place the eigenvalues of $\boldsymbol{A} - \boldsymbol{BF}$ arbitrarily? Why or why not?
   (b) 30 points. Find a state feedback control rule $\boldsymbol{F}$ such that the eigenvalues of the system with state feedback are at -1 and -2.
   (c) 10 points. Is $\boldsymbol{F}$ unique?
   (d) 30 points. Discuss the stability properties of this system with state feedback.
   (e) 10 points. Is the system with state feedback reachable?
   (f) 10 points. Is the system with state feedback observable?
   (g) 10 points. Is the system with state feedback minimal?

5. 80 points total. Given the same system as in the previous problem, suppose you build a compensated auxiliary system with state $\boldsymbol{w}(t)$ such that

$$\dot{\boldsymbol{w}}(t) = \boldsymbol{A}\boldsymbol{w}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{L}\left(\boldsymbol{y}(t) - \boldsymbol{C}\boldsymbol{w}(t) - \boldsymbol{D}\boldsymbol{u}(t)\right)$$

   where $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}\}$ are the same as in the previous problem.

   (a) 20 points. Is it possible to design a compensator $\boldsymbol{L}$ such that that the eigenvalues of $\boldsymbol{A} - \boldsymbol{LC}$ can be placed arbitrarily?
   (b) 40 points. Discuss the behavior of $\|\boldsymbol{x}(t) - \boldsymbol{w}(t)\|^2$ as $t \to \infty$ for the compensator $\boldsymbol{L} = [3, 0]^\top$.
   (c) 20 points. Compute the transfer function of the system with the state estimator $\boldsymbol{L} = [3, 0]^\top$ and your state feedback control rule $\boldsymbol{F}$ from the previous problem.