ECE504 Homework Assignment Number 10
Due by 8:45pm on 08-Dec-2009

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 7 points total. Consider the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$ and $\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t)$ where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$$

   (a) 2 points. Determine an appropriate linear state feedback law of the form $\boldsymbol{u}(t) = \boldsymbol{v}(t) - \boldsymbol{F}\boldsymbol{x}(t)$ so that the closed-loop transfer function has poles at -2, -2, and -3.

   (b) 1 point. Find the closed-loop transfer function of the system.

   (c) 2 points. Find the appropriate input gain p such that the closed-loop system tracks a step input $\boldsymbol{v}(t) = c\mathbb{I}(t)$.

   (d) 2 points. Is the closed-loop system controllable? Is the closed-loop system observable?

2. 4 pts. Given the system

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & -1 \end{bmatrix} \boldsymbol{x}(t) \end{aligned}$$

   (a) Find the state estimator $\boldsymbol{L}$ such that the eigenvalues of $(\boldsymbol{A} - \boldsymbol{L}\boldsymbol{C})$ are both at $-\alpha$ for some positive $\alpha \in \mathbb{R}$. What can you say about the entries of of $\boldsymbol{L}$ when α is large?

   (b) Suppose that the output has a small constant offset ϵ; that is, $y(t) = [1\ -1]\boldsymbol{x}(t) + \epsilon$. Assuming the state estimator's output is $\boldsymbol{w}(t)$, describe the behavior of the state estimate error vector $\boldsymbol{x}(t) - \boldsymbol{w}(t)$ for all t and as $t \to \infty$.

3. 4 points. You are given the continuous time state-space system described by

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \boldsymbol{D} = 0,$$

   Suppose you design an *uncompensated* state estimator such that

$$\dot{\boldsymbol{w}}(t) = \tilde{\boldsymbol{A}}\boldsymbol{w}(t) + \boldsymbol{B}u(t).$$

   where you are given an imperfect $\boldsymbol{A}$ matrix

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} 0 & 1+\epsilon \\ 0 & 0 \end{bmatrix}$$

   Describe the behavior of $\boldsymbol{x}(t) - \boldsymbol{w}(t)$ for the uncompensated estimator for all $t \geq 0$ in terms of the input $u(t)$ for all $t \geq 0$ and the initial states $\boldsymbol{x}(0)$ and $\boldsymbol{w}(0)$.