ECE504 Homework Assignment Number 2
Due by 8:45pm on 22-Sep-2009

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 5 points. Chen 2.19. Be sure to explicitly define your state in your solution.

2. 7 points total. Suppose you have a discrete time system with input-output relationship

$$y[k] = \sum_{n=0}^{N-1} \lambda^n u[k-n]$$

for some real constant $0 < \lambda < 1$.

   (a) 2 points. Let $N = 3$ and let the state $x(k) = [u(k-1)\ u(k-2)]^\top$. Find the matrices $\boldsymbol{A}(k)$, $\boldsymbol{B}(k)$, $\boldsymbol{C}(k)$, and $\boldsymbol{D}(k)$ such that

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)u(k) \\ y(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)u(k). \end{aligned}$$

   (b) 2 points. Let $N = 3$ and let the state

$$x(k) = \begin{bmatrix} u(k-1) + u(k-2) \\ u(k-1) \end{bmatrix}$$

   Find the matrices $\boldsymbol{A}(k)$, $\boldsymbol{B}(k)$, $\boldsymbol{C}(k)$, and $\boldsymbol{D}(k)$ such that

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)u(k) \\ y(k) &= \boldsymbol{C}(k)\boldsymbol{x}(k) + \boldsymbol{D}(k)u(k). \end{aligned}$$

   (c) 1 point. Comment on the uniqueness of the state.

   (d) 2 points. Suppose

$$y[k] = \sum_{n=0}^{N-1} \lambda^n u[k-n] - y[k-1].$$

   Select an appropriate state and explicitly write the state-space description of this system for the case $N = 3$.

3. 3 points. Using the adjoint/determinant method discussed in lecture, find the inverse of this matrix

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & c \\ 0 & d & e \end{bmatrix}$$

where all terms in the matrix are real-valued. Under what conditions does the matrix inverse exist?