ECE504 Homework Assignment Number 3
Due by 8:45pm on 29-Sep-2009

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 4 points. Find two different state-space descriptions for the discrete time system

$$\hat{g}(z) = \frac{z^{-2} + 5z^{-1} - 4}{z^{-2} + 2z^{-1} - 3}.$$

   Confirm that both of your state space descriptions give the same transfer function.

2. 5 points. Repeat Chen problem 2.19 except let the capacitance of the capacitor across the output be $C(t)$, i.e. this capacitor is time varying. Compute a state space description. Compute an input/output differential equation. Can you compute a transfer function?

3. 6 points total. Suppose you have a non-linear continuous-time system with state space description

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ a + bu(t)/x_3(t) \\ u(t) \end{bmatrix}$$

   with $y(t) = x_1(t)$.

   (a) 2 points. Given a constant nominal input $u(t) = \tilde{u}(t) \equiv u_0 < 0$ for all $t \geq 0$, confirm that

$$\tilde{\boldsymbol{x}}(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \\ \tilde{x}_3(t) \end{bmatrix} = \begin{bmatrix} at^2/2 + (bm_0/u_0)[(1 + u_0t/m_0)\ln(1 + u_0t/m_0) - u_0t/m_0] \\ at + b\ln(1 + u_0t/m_0) \\ m_0 + u_0t \end{bmatrix}$$

   is a nominal solution to the state dynamics equation, where $m_0 = x_3(0)$.

   (b) 4 points. Linearize this system at this nominal solution and write the linearized state-space description.