HW4 corrected solution

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$$\dot{x}(t) = \begin{bmatrix} -\sin t & 0 \\ 0 & -\cos t \end{bmatrix} x(t)$$

$$\left.\begin{aligned} \dot{x}_1(t) &= -\sin t \, x_1(t) \Rightarrow x_1(t) = a e^{\cos t} \\ \dot{x}_2(t) &= -\cos t \, x_2(t) \Rightarrow x_2(t) = b e^{-\sin t} \end{aligned}\right\}$$ a and b are chosen to satisfy the initial conditions

Fundamental matrix method ...

$\underline{x}_1(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow a = 1, \ b = 0$ ← this is incorrect. If we, however, choose $\underline{x}_1(0) = \begin{bmatrix} e \\ 0 \end{bmatrix}$, this initial state is linearly independent of $\underline{x}_2(0)$ and implies $a=1, b=0$

$\underline{x}_2(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow a = 0, \ b = 1$

check: $1 e^{\cos 0} = 1 e^{1} = e$ ✓

$0 e^{-\sin 0} = 0$ ✓

so the fundamental matrix is

$$\underline{\underline{X}}(t) = \begin{bmatrix} e^{\cos t} & 0 \\ 0 & e^{-\sin t} \end{bmatrix}$$

$$\underline{\underline{X}}^{-1}(s) = \begin{bmatrix} e^{-\cos s} & 0 \\ 0 & e^{+\sin s} \end{bmatrix}$$

STM: $$\boxed{\Phi(t,s) = X(t) X^{-1}(s) = \begin{bmatrix} e^{\cos t - \cos s} & 0 \\ 0 & e^{\sin s - \sin t} \end{bmatrix}}$$