ECE504 Homework Assignment Number 5
Due by 8:45pm on 13-Oct-2009

Tips: Make sure your reasoning and work are clear to receive full credit for each problem.

1. 3 pts. For each matrix $\boldsymbol{A}$ below, find its characteristic polynomial, its eigenvalues, their algebraic multiplicities, bases for all of the eigenspaces, and the eigenvalues' geometric multiplicities. Comment on diagonalizability.

   (a)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

   (b)

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

   (c) $\boldsymbol{A} = \boldsymbol{I}_n$

2. 4 points. Compute $e^{t\boldsymbol{A}}$ and $\boldsymbol{A}^{100}$ for both $\boldsymbol{A}$ matrices in Problem 1, parts (a) and (b).

3. 4 pts. Suppose that $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t)$, $t \in \mathbb{R}$, and that $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$ is a *constant* matrix. You do a zero-input response experiment and find that when $\boldsymbol{x}(0) = [1, 1]^\top$, $\boldsymbol{x}(t) = [2e^{-2t} - e^{-3t}, e^{-3t}]^\top$ for all $t \in \mathbb{R}$. You do another zero-input response experiment and find that when $\boldsymbol{x}(0) = [1, -1]^\top$, $\boldsymbol{x}(t) = [e^{-3t}, -e^{-3t}]^\top$ for all $t \in \mathbb{R}$. Find $\boldsymbol{A}$ and the state transition matrix.

4. 4 points. Chen 4.2.